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PB80-169436

# Scheme for Estimating Dispersion Parameters as a Function of Release Height

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| | |
|---|---|
| 1. REPORT NO. EPA-600/4-79-062 | 2. |
| 3. RECIPIENT'S ACCESSION NO. PB 80 169436 | |
| 4. TITLE AND SUBTITLE<br>SCHEME FOR ESTIMATING DISPERSION PARAMETERS AS A FUNCTION OF RELEASE HEIGHT | 5. REPORT DATE<br>October 1979 |
| | 6. PERFORMING ORGANIZATION CODE |
| 7. AUTHOR(S)<br>John S. Irwin | 8. PERFORMING ORGANIZATION REPORT NO. |
| 9. PERFORMING ORGANIZATION NAME AND ADDRESS<br>Same as block 12 | 10. PROGRAM ELEMENT NO.<br>1AA603A AB-025 (FY-79) |
| | 11. CONTRACT/GRANT NO. |
| 12. SPONSORING AGENCY NAME AND ADDRESS<br>Environmental Sciences Research Laboratory - RTP, NC<br>Office of Research and Development<br>U.S. Environmental Protection Agency<br>Research Triangle Park, North Carolina 27711 | 13. TYPE OF REPORT AND PERIOD COVERED<br>In house 7/77 - 6/79 |
| | 14. SPONSORING AGENCY CODE<br>EPA/600/09 |

15. SUPPLEMENTARY NOTES

16. ABSTRACT

Based on an investigation where the dispersion parameters are assumed to have the form $\sigma_{z,y} = \sigma_{w,v}\, t\, F_{z,y}$, a generalized scheme is presented for estimating the dispersion parameters as a function of release height. Further development is needed to refine the scheme for more generalized applicability, since, as documented in this discussion, the scheme requires as input meteorological data not routinely available. The scheme incorporates results from various studies, and once it is more practically structured it will prove useful for characterizing dispersion from tall sources in a variety of situations. The generalized scheme was developed particularly for Gaussian plume modeling; therefore, it is restricted to modeling applications having flat terrain and having steady-state meteorological conditions.

17. KEY WORDS AND DOCUMENT ANALYSIS

| a. DESCRIPTORS | b. IDENTIFIERS/OPEN ENDED TERMS | c. COSATI Field/Group |
|---|---|---|
| * Air pollution<br>* Meteorology<br>* Atmospheric diffusion<br>* Estimating<br>* Mathematical Models<br>* Height | | 13B<br>04B<br>04A<br>12A |

| | | |
|---|---|---|
| 18. DISTRIBUTION STATEMENT<br>RELEASE TO PUBLIC | 19. SECURITY CLASS (This Report)<br>UNCLASSIFIED | 21. NO. OF PAGES<br>68 |
| | 20. SECURITY CLASS (This page)<br>UNCLASSIFIED | 22. PRICE |

EPA Form 2220-1 (9-73)

United States
Environmental Protection
Agency

Environmental Sciences Research
Laboratory
Research Triangle Park NC 27711

EPA-600/4-79-062
October 1979

Research and Development

PB80-169436

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EPA-600/4-79-062
October 1979

# SCHEME FOR ESTIMATING DISPERSION PARAMETERS AS A FUNCTION OF RELEASE HEIGHT

by

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ENVRIONMENTAL SCIENCES RESEARCH LABORATORY
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## AFFILIATION

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## FOREWORD

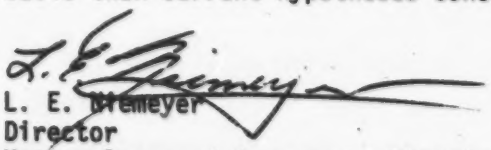
As discussed by Mr. Ken Calder the increasing concern of the last decade in environmental issues, and the fuller appreciation that air quality simulation modeling may provide a basis for the objective management of air quality, has generated an unprecedented inter est in the development of techniques for relating air quality and pollutant emissions through appropriate modeling of the atmospheric transport and dispersion processes that are involved. A multitude of recent publications testify to the widespread acceptance of meteorological-type air quality modeling as an important rational basis for air quality management.

Earlier attitudes towards the quantitative estimation of atmospheric dispersion of windborne material from industrial and other sources were strongly influenced by a Gaussian-plume dispersion parameter scheme introduced in 1958, and published in 1961, by Dr. F. Pasquill of the Meteorological Office, United Kingdom. In spite of the gradual appearance in recent years of air quality models based on more sophisticated formulations of the atmospheric processes, great use continues to be made of the simpler Gaussian-plume models. As a direct consequence of the unprecedented interest in the subject, a Workshop on Stability Classification Schemes and Sigma Curves was held at the American Meteorological Society Headquarters in Boston on 27-29 June 1977.

About 25 scientists representing a cross-section of regulatory agencies, public utilities, universities, research laboratories, and consulting corporations were invited by the AMS to the workshop. One of the conclusions reached at the workshop was that the present characterizations of steady-state dispersion over flat, homogeneous sites represent an unfinished state of the art. They suggested that examination of dispersion data in terms of parameters now recognized as important might resolve many of the differences so evident between presently available Sigma schemes. However, they recognized that more observations are required in order to confirm existing hypotheses of the more appropriate para-

meterization of dispersion from elevated releases.

This report presents one such investigation in which an attempt was made to synthesize existing dispersion data into one generalized scheme using currently hypothesized parameters of interest. The resulting generalized scheme was developed especially with tall sources in mind but needs further refinement and confirmation before it can be used in routine assessments of air quality. In this regard, the scheme (or some simplification thereof) does provide a framework for the analysis of future field data. It is perhaps unnecessary to emphasize that the author resoundly corroborates the workshop participants' recommendation for more field data of dispersion from elevated releases accompanied by comprehensive meteorological measurements. Such data is deemed necessary for the development of generalized scheme on a more sure basis than current hypotheses concerning the parameterization of dispersion.



L. E. Niemeyer
Director
Meteorology and Assessment Division
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Research Triangle Park, North Carolina
March 1979

## ABSTRACT

Recent concern about properly characterizing the Gaussian plume dispersion parameters as a function of release height and surface roughness prompted the American Meteorological Society (AMS) to sponsor a workshop on stability classification schemes and sigma curves. Since sufficient field data was lacking, a generalized scheme was not recommended at the workshop.

Based on an investigation where the dispersion parameters are assumed to have the form $\sigma_{z,y} = \sigma_{w,v}\, t\, F_{z,y}$, a generalized scheme is presented for estimating the dispersion parameters as a function of release height. Further development is needed to refine the scheme for more generalized applicability, since, as documented in this discussion, the scheme requires as input meteorological data not routinely available. The scheme incorporates results from various studies, and once it is more practically structured it will prove useful for characterizing dispersion from tall soruces in a variety of situations. The generalized scheme was developed particularly for Gaussian plume modeling; therefore, it is restricted to modeling applications having flat terrain and having steady-state meteorological conditions.

This report covers a period from July 15, 1977 to March 15, 1979 and work was completed as of June 15, 1979.

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## LIST OF SYMBOLS

| | |
|---|---|
| $h$ | -- the depth of the convectively mixed layer |
| $h_s$ | -- the release height (stack height) of the emissions |
| $\Delta h$ | -- the plume rise of emissions |
| $\kappa$ | -- the von-Karman constant, 0.4 |
| $L$ | -- $\overline{\theta}\, u_*^3/(k\, g\, \overline{w'\theta'})$, the Monin-Obukhov scaling length where g is the acceleration due to gravity and $\overline{w'\theta'}$ is the mean surface kinematic heat flux |
| $t$ | -- the travel time downwind of the release position |
| $u$ | -- the horizontal wind speed in the direction of pollutant transport |
| $u_*$ | -- the surface friction velocity |
| $w_*$ | -- the convective velocity scale |
| $x,y,z$ | -- rectangular coordinates; x along the mean wind, y across mean wind, z vertical. Origin located at ground level at release position |
| $H_e$ | -- $h_s + \Delta h$, the effective release height |
| $\Delta\theta$ | -- the horizontal wind direction shear through the vertical extent of the plume |
| $\overline{\theta}$ | -- the mean potential temperature |
| $\sigma_{w,v}$ | -- the standard deviations of the vertical and lateral wind component fluctuations |
| $\sigma_{z,y}$ | -- the standard deviation of the vertical and lateral pollutant distributions |

## ACKNOWLEDGMENT

The author is especially indebted to Dr. Francis Binkowski and Dr. Robert Lamb, whose knowledge of dispersion processes and of atmospheric turbulence structure were essential to the completion of this study. Both provided perceptive counsel, and both generously donated research results and data. The author is also grateful for Mr. D. Bruce Turner's advice throughout the study. Finally, my thanks go to Pam Hinton and Joan Emory for their secretarial assistance in the preparation of this report.

# SECTION 1

# INTRODUCTION

The following discussion documents the basis of a practical scheme to estimate dispersion parameters that would be used in a Gaussian plume model for estimating the surface concentrations of a pollutant emitted at an elevated height. In a summary review of the workshop sponsored by the American Meteorological Society on Stability Classification Schemes and Sigma Curves, Hanna et al. (1977) suggest that a generalized scheme, explicitly handling most of the processes affecting dispersion, might promote more consistent modeling than presently exists and might provide more insight than presently exists for directing further research. Workshop participants noted that several schemes are currently used to estimate the dispersion of material as a function of downwind transport. Each scheme has its inherent limitations because of the data base used to construct the estimation scheme. Reviewed at the AMS workshop were: (1) the data bases of current estimation schemes and (2) several current rules of thumb that account for dispersion processes not explicitly ascribable to the current schemes, such as surface roughness and buoyant plume rise. Even when properly applied, the schemes and rules of thumb represent an unfinished state-of-the-art.

Using the AMS workshop recommendations as a basis, a generalized scheme was constructed. In the development and construction of the generalized scheme it became apparent that more information would be needed in order to complete the characterization of dispersion. Hence, the generalized scheme was designed to facilitate future refinement and improvement through the incorporation of new field data and new research results. The scheme views dispersive processes such as buoyant plume rise, as independent of the other ongoing dispersive processes. With this viewpoint, the total dispersion is then the summation of the individual effects of each process (p. 207, Högström [1964] and p. 23, Pasquill [1976]).

Hence, the total vertical dispersion $\sigma_z$ becomes:

$$\sigma_z^2 = \sigma_{zo}^2 + \sigma_z'^2 \qquad (1)$$

where $\sigma_{zo}$ = vertical dispersion due to atmospheric turbulence

$\sigma_z'$ = induced vertical dispersion due to buoyancy effects

Currently, $\sigma_{zo}$ is viewed as a function of the surface roughness, of the effective release height, of a surface stability parameter (such as the Monin-Obukhov scaling length), and of the depth of the convectively mixed layer (which represents the upper limit of vertical dispersion during unstable conditions). Also, $\sigma_{zo}$ is affected by urban "heat island" effects; however, the degree of urban effects likely depend on the size and layout of the city, and further studies are needed to predict/estimate urban effects in a generalized manner.

The total lateral dispersion, $\sigma_y$, becomes:

$$\sigma_y^2 = \sigma_{yo}^2 + \sigma_y'^2 + \sigma_y''^2 \qquad (2)$$

where $\sigma_{yo}$ = lateral dispersion due to atmospheric turbulence

$\sigma_y'$ = induced lateral dispersion due to buoyancy effects

$\sigma_y''$ = induced lateral dispersion due to the turning with height of the horizontal wind direction

Currently, $\sigma_{yo}$ is viewed as a function of a surface stability parameter (such as the Monin-Obukhov scaling length), of the depth of the convectively mixed layer, and of the lateral velocity fluctuations (which are a function of averaging time, of stability, and of terrain characteristics).

Sections 2 and 3 review the various techniques mentioned at the AMS workshop for modeling the vertical and lateral dispersion as expressed by Equations 1 and 2. The terms within Equations 1 and 2 are discussed individually in order to explain the rationale for the particular modeling techniques selected for use in the construction of the generalized scheme. Section 4 summarizes the generalized scheme and presents an overview of the scheme in its basic form. Section 5 concludes the discussion with an outline of the analysis required to cast the scheme in a form practical for routine use.

## SECTION 2

## THEORY AND DEVELOPMENT-

## VERTICAL DISPERSION

### ESTIMATION OF $\sigma_{zo}$

Pasquill (1971) derived from Taylor's (1921) equation an expression for estimating the vertical dispersion (neglecting buoyancy) $\sigma_{zo}$ which Draxler (1976) investigated in the following form:

$$\sigma_{zo} = \sigma_w \, t \, F_z(t/t_L) \qquad (3)$$

where $\sigma_w$ = total standard deviation of the vertical wind component fluctuations calculated or estimated over a duration of about 1 hour

$t$ = the travel time downwind of the release point

$t_L$ = the Lagrangian time scale

For $F_z$ to be a universal function, independent of stability and effective release height, and to satisfy Taylor's limits for small and large travel times, conditions must be stationary and homogeneous both vertically and horizontally. With such conditions, $\sigma_{zo}$ would be proportional to $\sqrt{t}$ at large travel times. In reality the vertical turbulence structure is not homogeneous. In discussing this point, Pasquill (1971) notes that for small travel times the effects of nonhomogeneity are minimal and $\sigma_z$ should still be proportional to t. But for large travel times the effects of nonhomogeneity can not be neglected, and $F_z$ becomes a function of release height and stability as well as travel time. Furthermore, it is reasonable to expect $\sigma_{zo}$ to be proportional to something other than $\sqrt{t}$ for large travel

times. For these reasons, as recommended at the AMS workshop, the following parameters are needed to characterize $\sigma_{zo}$:

$Z_o$ = surface roughness length

$H_e = h_s + \Delta h$, the effective release height, where $h_s$ is the stack height and $\Delta h$ is the buoyant plume rise

$L$ = Monin-Obukhov length scale (a stability parameter)

$h$ = depth of the convectively mixed layer

The results of similarity theory suggest that $\sigma_w$ can be envisioned as a function of $H_e$, L, and $Z_o$ (see Binkowski, 1979), where $H_e$ is restricted to heights within the surface layer where $u_*$ (the surface friction velocity) is constant. Studies by Kaimal et al. (1976) and by Willis and Deardorff (1974) suggest that $\sigma_w$ is a function of h, L, $H_e$, and $u_*$, where $H_e$ is now within the convectively mixed layer. Combining these results with the characterizations of $F_z$ by Draxler (1976) and by Nieuwstadt and Van Duuren (1979) presents much evidence that $\sigma_{zo}$ can be considered functionally as:

$$\sigma_{zo} = \sigma_w(H_e, L, Z_o, h, u_*)\ F_z(H_e, L, h, u_*, t_L, t) \qquad (4)$$

$H_e$, L, h, and $u_*$ seem necessary to characterize $F_z$, because the basic assumptions of homogeneity used to develop Equation 3 are in practice violated. In simplistic terms, $F_z$ describes the growth of $\sigma_{zo}$ as a function of downwind transport and responds to the overall structure of the turbulence; whereas $\sigma_w$ describes the local features of the problem, assuming the structure of turbulence remains fixed over the transport downwind.

Within the context of this discussion, the various schemes for estimating dispersion can be examined as special cases of Equation 4. In the case of the Pasquill-Gifford (P-G) scheme (Turner, 1970), values for $H_e$ and $Z_o$ have been specified as surface release height and 3 cm, respectively; however, values for h, L, and $u_*$ are not available. Similarly, the Brookhaven (BNL) scheme (Smith, 1968) values for $H_e$ and $Z_o$ are 100 m and 1 m, respectively; however, values for h, L, and $u_*$ are not available. Conceivably, rules of thumb could be employed to adjust these schemes for other modeling situations, but in practice many questions arise. For instance, a description of the

effects of surface roughness on vertical dispersion was developed by F. B. Smith (1972) using K-theory. These results are for surface releases and address only vertical dispersion. Hence, these results are questionable if applied to the BNL scheme. Furthermore, to handle release heights other than at surface and 100 m using these two schemes: (1) the schemes would individually have to be adjusted to the particular situation with respect to L, $Z_o$, h, $u_*$, and $t_L$, and (2) adjustment would have to be made for $H_e$. The required adjustment would be very difficult since most of the original values of the parameters are unknown. In addition, the appropriate manner to extrapolate to other release heights is unclear. Because of the many uncertainties involved, most current schemes for estimating dispersion are used unadjusted.

Considering these uncertainties, it was decided to characterize $\sigma_{zo}$ in some other manner than by tailoring existing schemes (such as the BNL or P-G schemes). Pasquill's (1978) review of the various theoretical models characterizing dispersion suggests that each of these models is valid for only a limited range of conditions. However, these models do provide valuable results, and even their piecemeal inclusion is important in formulating a generalized model. Hence, investigation began into further development of existing characterization of $F_z$, using results gained from laboratory studies and theoretical characterizations of dispersion. Using Equation 4 to characterize $\sigma_{zo}$ allows most of the factors affecting $\sigma_{zo}$ to be handled explicitly; see Draxler (1976) and Nieuwstadt and Van Duuren (1979). And, as shown by Nieuwstadt and Van Duuren, this basic model lends itself to ready adaptation and improvement as data become available from new field studies. Furthermore, Cramer (1976) reported the utility of characterizing $\sigma_{zo}$ for elevated releases in this manner. Cramer used a very simple characterization of the terms in Equation 4. A power-law relationship was used to characterize $\sigma_w$ as a function of height where the power-law exponent was varied as a function of stability and $F_z$ was set to a constant value of one. Hence, Draxler's results and Nieuwstadt and Van Duuren's results represent refinements to an already proven modeling technique. The fact that Cramer's model performed as well as it did, suggest that for elevated releases $F_z$ may not have to be specified in detail. This mitigates to some extent the concern expressed at the AMS workshop regarding the routine use of models using characterizations of $F_z$ before a consensus as to the detailed form of $F_z$ has been reached.

The following discussion describes the characterization of $F_z$ and is divided by stability into two parts - unstable conditions and neutral-to-stable conditions. The division was made because the set of parameters found useful for characterizing $F_z$ during unstable conditions was found to be different than the set of parameters found useful for neutral and stable conditions.

Unstable Conditions

Although Draxler (1976) did specify a form for $F_z$ for both elevated and near-surface releases during unstable conditions, he did so on a limited data base. The Porton data, Hay and Pasquill (1957), were the only data for elevated releases providing direct measurements of both the vertical dispersion and the standard deviation of the elevation angle $\sigma_e$. Unfortunately, other extensive sets of field data for possibly improving Draxler's results do not exist. However, laboratory data do exist that contain direct measurements of the vertical dispersion during convectively unstable conditions, resulting from the tank experiments performed by Willis and Deardorff (1976, 1978). Their data were for scaled release heights, $H_e/h$, of 0.067 and 0.24, where $H_e$ was the actual release height since $\Delta h$ was zero. To these data can be added the numerical results reported by Lamb (1978) for convectively unstable conditions. The data presented by Lamb were for a scaled release height of 0.26 where $\Delta h$ was zero. Lamb (1979) also has results for scaled release heights of 0.025, 0.50, and 0.71 where $\Delta h$ was zero. The Porton data, the tank data, and the numerical data are listed in Appendix A.

For the Porton data, h was assumed to be 1000 m. Values of $F_z$ were computed as $\sigma_z/(\sigma_w t)$ and depicted graphically as a function of $t^* = w_* t/h$, where $w_*$ is the convective velocity scale and is equal to:

$$w_* = u_*( -h/(k\,L) )^{1/3} \qquad (5)$$

In Equation 5, k is the von Karman constant. The results are depicted separately in Figures 1 and 2, since the data seem to suggest a different variation for $F_z$ as a function of $t^*$ for releases where $H_e/h$ was less than or equal to 0.67 (Figure 2) from the variation suggested for releases where $H_e/h$ was greater than or equal to 0.15 (Figure 1). The curves shown on Figures 1 and 2 are hand-drawn best fits to the trend reflected in the data values. For $t^*$ greater than 1, the distribution of the material by dispersion is effectively becoming

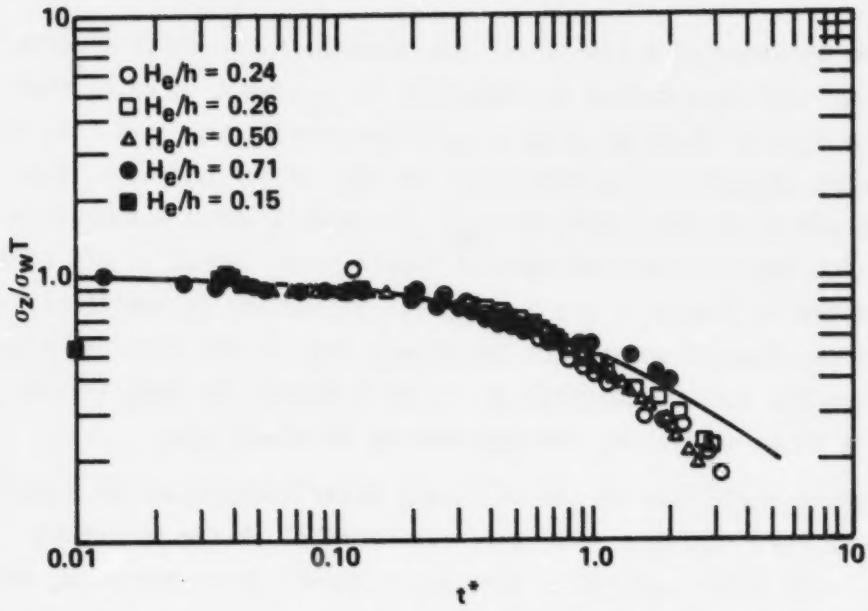


Figure 1. Values of $F_z$ versus t* for elevated releases during convectively unstable conditions.

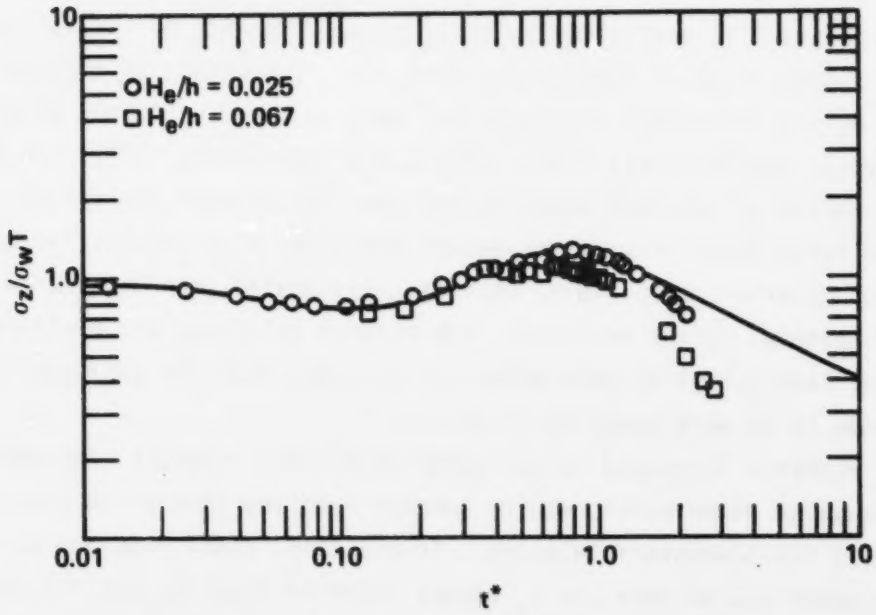


Figure 2. $F_z$ versus t* for near-surface releases during convectively unstable conditions.

limited by the presence of a finite vertical domain, h, in which dispersion can take place. The best method to characterize $\sigma_{zo}$ for $t^*$ greater than 1 depends on the type of Gaussian plume dispersion model to be used. If the dispersion model assumes reflections occur at the surface and at h, then no limit need be placed on the growth of $\sigma_{zo}$. For models which assume such multiple reflections, a continued rate of growth proportional to $\sqrt{t^*}$ could be used, as shown in Figures 1 and 2. Such an assumption is admittedly arbitrary, since we would not expect the growth rate of the vertical dispersion to obey the limits suggested by Taylor's theory for long travel times, a point discussed in the introduction of this section.

The standard deviations of the pollutant distribution used in developing Figures 1 and 2 were computed using vertical profiles of the pollutant distribution. For these particular data sets, $\Delta h$ was zero; hence, $\sigma_z$ was assumed to be equal to $\sigma_{zo}$. The height used in the computation of $\sigma_z$ from the profiles was the release height. As long as the distributions are Gaussian, we can use Figures 1 and 2 to estimate $\sigma_{zo}$ and use these estimates directly in a Gaussian plume model to estimate concentrations.

During convectively unstable conditions, the vertical dispersion of the pollutant material is controlled by the vigorous updrafts of the hot thermals and by the broad areas of slowly decending air. The effect of this convective structure on the turbulence structure has been reported by Kaimal et al. (1976), Willis and Deardorff (1976, 1978), and Lamb (1978, 1979). In brief, the distribution of vertical velocity fluctuations at most any height within the convectively mixed mixed layer is not Gaussian; as a result, for nonbuoyant releases of material, Lamb (1979) reports a systematic, non-Gaussian distribution of material in the vertical. For buoyant releases, the preliminary findings by Lamb (personal communication) indicate that the pollutant distribution tends to be more Gaussian in nature.

More research is needed to determine if buoyant releases when modeled using a Gaussian plume model require special handling similar to that required (Lamb, 1979) for nonbuoyant releases. In any case, these findings do raise questions about how to best use $\sigma_z$ values inferred from the surface concentration pattern by employing a Gaussian plume assumption. Some field studies have

characterized the vertical dispersion based on surface measurements by assuming the distribution in the vertical to be Gaussian. If, as Lamb's results suggest, the departure from a Gaussian distribution in the vertical is a function of the source characteristics, then characterizations of vertical dispersion from surface observations can be used directly only for sources having like characteristics as those used in the field study. To generalize such results to handle other source types would require rarely available data concerning the true vertical distribution taking place during the field study. For this reason, the caracterization of $F_z$ has been based solely on $\sigma_z$ values measured directly. If future studies indicate adjustments are needed in order to correct for non-Gaussian pollutant distribution, these adjustments can be made within the Gaussian plume model as Lamb suggests without altering the characterization of $F_z$. Another benefit of characterizing $F_z$ from $\sigma_z$ values measured directly is that the characterization of $F_z$ can be improved using field experiments without having to invoke any assumptions regarding the vertical distribution of material.

Neutral and Stable Conditions

For stable and neutral conditions, the Porton data were the only field data used by Draxler that contained direct measurements of both the vertical dispersion and the standard deviation of the elevation angle $\sigma_e$. Of the other field data used by Draxler, several cases of near-neutral data presented by Högström (1964) and by Hilst and Simpson (1958) were adapted for use in this study by assuming that conditions were neutral and estimating $\sigma_e$. In addition to these field data, a portion of the second-order closure results of the vertical dispersion versus downwind travel time reported by Lewellen and Teske (1975) were used. Lewellen and Teske depicted their profiles of turbulence during neutral conditions; therefore, their estimates of the vertical dispersion were usable for a surface release and an elevated release in this study. The second-order closure results and the mentioned field data are listed in Appendix A.

Values of $F_z = \sigma_{zo}/(\sigma_e x)$, where x is the downwind distance, were plotted as a function of travel time from which the time $T_o$ (when $F_z$ was approximately equal to 1/2) was subjectively determined. This type of analysis

seemed appropriate, considering Draxler's analysis. The small travel times (<40 s) of the Porton data did not prove useful in defining the variation of $F_z$ as a function of travel time. Hence, the Porton data were not used in succeeding analyses.

Figure 3 depicts the results of the above analysis, where $T_o$ has been plotted versus the release height of the data set from which $T_o$ was determined.

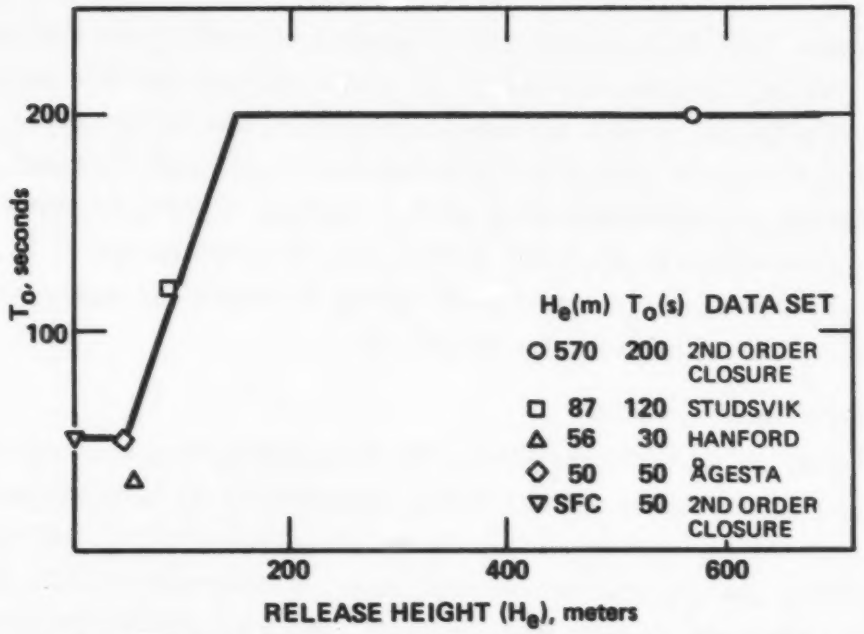


Figure 3. Characterization of the $T_o$ parameter for use in Draxler's equations for estimating dispersion during neutral and stable conditions.

Draxler estimated $T_o$ to be 50 seconds for a surface release and 100 seconds for an elevated release. The results depicted in Figure 3 appear consistent with Draxler's results, considering the release heights represented in his analysis. It would appear from the handful of data used in Figure 3 that $T_o$ is proportional to release height, being smallest for surface releases and largest for elevated releases. Many assumptions were made in developing Figure 3; hence, the variation of $T_o$ as a function of release height ($H_e$ in these cases, since $\Delta h$ was zero) was characterized simply as:

$$T_o \text{ (sec)} = \begin{cases} 50 & H_e \leq 50 \text{ m} \\ (3H_e - 50)/2 & 50\text{m} < H_e > 150 \text{ m} \\ 200 & H_e \leq 150 \text{ m} \end{cases} \quad (6)$$

Using this characterization of $T_o$, values of $F_z$ were plotted as a function of $t/T_o$ along with the Draxler (1976) characterizations of $F_z$ (Figure 4). The

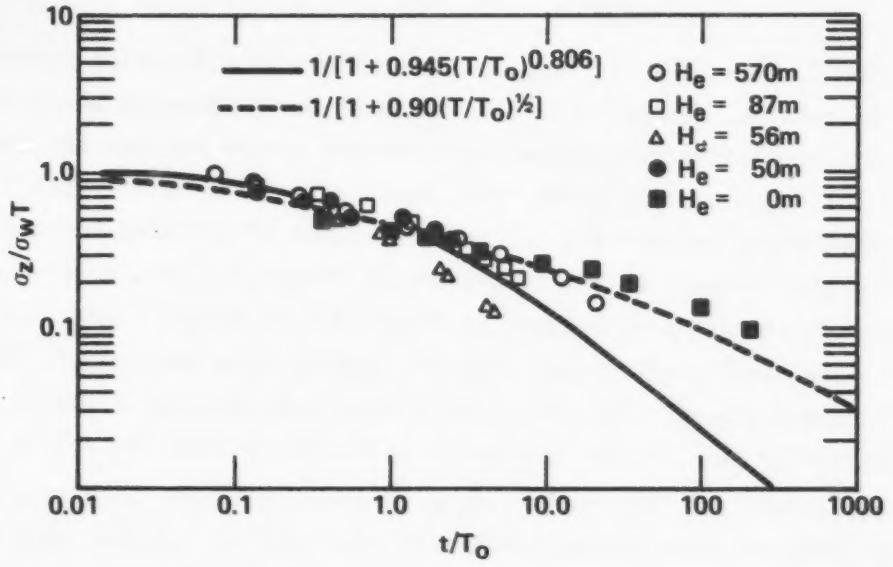


Figure 4. Values of $F_z$ versus $t/T_o$ for surface and elevated releases during neutral and stable conditions.

dashed line in Figure 4 is Draxler's recommendation for $F_z$ for surface releases:

$$F_z = 1/\{+ 0.9(t/T_o)^{1/2}\} \quad (7)$$

and the solid line is Draxler's recommendation for $F_z$ for elevated releases:

$$F_z = 1/\{1+ 0.945(t/T_o)^{0.806}\} \quad (8)$$

Draxler's (1976) characterization of $F_z$ for elevated releases, Equation 8, seems to perform well for effective release heights above 50 m. And his characterization for surface releases, Equation 7, seems to follow Lewellen and Teske's (1975) results for a surface release. All the data do not fit these characterizations, as some deviations can be seen. However, whether the

deviations occur because of data problems or because of characterization problems is difficult to determine. More data are needed before a complete characterization of $\sigma_{zo}$ during neutral and stable conditions can be developed.

ESTIMATION OF $\sigma_z'$

Pasquill (1976, 1979) gives an expression for estimating the induced dispersion due to buoyancy, $\sigma_z'$, as:

$$\sigma_z' = \Delta h/3.5 \qquad (9)$$

In Equation 9, $\Delta h$ equals the buoyant plume rise. In discussing buoyancy-induced dispersion, Pasquill (1974) notes that during buoyant plume rise strong evidence indicates a linear relationship exists between the instantaneous plume depth and the plume rise; however, no data base exists allowing direct comparisons between the induced dispersion of a rising buoyant plume and the natural dispersion and time-averaged dispersion of a passive plume. The linear relationship is evident in Figure 4.1 of Briggs (1969), which depicts plume depth versus plume rise as inferred from photographs taken at three TVA power plants. Pasquill (1974) notes that further evidence regarding the magnitude of the induced dispersion is available from Högström's (1964) measurements. Högström's data were developed from successively released puffs of smoke photographed from the point of release, making it possible to compare the instantaneous size and scatter of the puff centers. The TVA studies of plume dispersion and Högström's results suggest that the total induced plume depth due to buoyancy can be approximated as equal to the plume rise. Pasquill assumes the distribution of concentration within a buoyancy-dominated plume can be approximated as uniform. On this basis, the equivalent standard deviation becomes $\Delta h/(2\sqrt{3})$, which leads to the expression given in Equation 9.

## SECTION 3

## THEORY AND DEVELOPMENT-

## LATERAL DISPERSION

### ESTIMATION OF $\sigma_{yo}$

As a starting point the following formula was suggested (Hanna et al., 1977) for estimating the lateral dispersion (neglecting buoyancy and directional shear effects), $\sigma_{yo}$:

$$\sigma_{yo} = \sigma_v \; t \; F_y(t/t_L) \qquad (10)$$

In Equation 10, $\sigma_v$ is the standard deviation of the horizontal crosswind component of the wind. The estimate of $\sigma_v$ should have an averaging time comparable to the averaging time to be associated with the concentration estimate. The above expression for $\sigma_{yo}$ was derived by Pasquill (1971) from Taylor's (1921) equation and assumes that conditions are stationary and homogeneous, vertically and horizontally.

Various estimates of $F_y$ are available (e.g., Pasquill, 1976; Draxler, 1976; Deardorff and Willis, 1975). Even though different data bases have been used, the various estimates of $F_y$ are quite similar.

Recently Nieuwstadt and Van Duuren (1979) proposed a generalized form for $F_y$, valid for unstable conditions, which, in the limit of convectively unstable conditions (when $-h/L \to \infty$), approximates Deardorff and Willis's 1975 result:

$$\sigma_{yo}/h = 0.51 \; t^*/\{1+ 0.91 \; t^*\}^{\frac{1}{2}} \qquad (11)$$

$$t^* = w_* t/h$$

In the limit of neutral conditions (when $-h/L \rightarrow 0$), the form approximates Draxler's (1976) result:

$$\sigma_{yo} = \sigma_v \, t/\{1+ 0.9(t/T_o)^{\frac{1}{2}}\} \qquad (12)$$

$$T_o = 1000 \text{ s}$$

The appeal of Nieuwstadt and Van Duuren's form for $F_y$ is that it explicitly incorporates the scaling parameters appropriate for both convectively unstable conditions and neutral conditions.

Their proposed characterization of $F_y$ was:

$$F_y = 1/\{1+ t/T_i\}^{\frac{1}{2}} \qquad (13)$$

In Equation 13, $T_i^{-1} = (c \, u_*/h)\{1+ d(-h/L)\}^{1/3}$. They completed their characterization of $\sigma_{yo}$ by using a form for $\sigma_v$ similar to that proposed by Pasquill et al. (1977):

$$\sigma_v = a \, u_*\{1+ b(-h/L)\}^{1/3} \qquad (14)$$

The constants a, b, c, and d were solved for by constraining $\sigma_{yo}$ to conform to Deardorff and Willis's result when $-h/L \rightarrow \infty$ and constraining $\sigma_{yo}$ to approximate Draxler's result when $-h/L \rightarrow 0$. They used Tennekes's (1969) relationship to estimate the boundary layer height in the neutral limit as:

$$h = 0.3 \, u_*/f \qquad (15)$$

In Equation 15, the Coriolis parameter f was estimated to be $10^{-4} s^{-1}$.

Nieuwstadt and Van Duuren compared estimates of $\sigma_{yo}$ using their generalized characterization of $\sigma_v$ and $F_y$ with lateral dispersion measurements from six tracer experiments performed at Cabauw in the Netherlands. These data were for conditions between convectively unstable conditions and neutral conditions, when $0 < -h/L < 15$ and the horizontal wind speed at release height (generally 200 m) varied between 4 and 10 m/s. The few cases of field data available compared quite well with their estimates of $\sigma_{yo}$.

More for cosmetic than technical reasons, a minor embellishment to the Nieuwstadt and Van Duuren characterization of $F_y$ is suggested. Since the forms of Equations 11 and 12 differ, it is suggested that Deardorff and Willis's result be approximated as:

$$\sigma_{yo}/h = 0.51\ t^*/\{1+ 0.45\ t^{*\frac{1}{2}}\} \qquad (16)$$

and allow $F_y$ to have the form:

$$F_y = 1/\{1+ (t/T_i)^{\frac{1}{2}}\} \qquad (17)$$

Now Equations 12, 16, and 17 have the same form. This allows the constants a, b, c, and d to be specified such that in the convective limit of $-h/L \rightarrow \infty$, $\sigma_{yo}$ will equal Equation 16 and in the neutral limit of $-h/L \rightarrow 0$, $\sigma_{yo}$ will equal Equation 12.

In solving for the constants, the neutral value for $\sigma_v/u_*$ of 1.78 suggested by Binkowski (1979) can be employed, in which case the von Karman constant, k, should be 0.4. According to Nieuwstadt and Van Duuren, (1979) Equation 15 is used to estimate h in the limit as $-h/L \rightarrow 0$. The procedure to solve for the constants is the same as outlined by Nieuwstadt and Van Duuren and results in:

$$a = 1.78$$
$$b = 0.059$$
$$c = 2.5$$
$$d = 0.00133$$

The preceding discussion outlined a characterization of $\sigma_{yo}$ for unstable conditions that, as conditions become neutral, matches Draxler's characterization for $\sigma_{yo}$ valid for neutral and stable conditions. As with the characterization in Section 2 for $\sigma_{zo}$, the characterization presented for $\sigma_{yo}$ explicitly handles many of the parameters considered important for lateral dispersion. By using data from studies in which these parameters are known or measured, much as Nieuwstadt and Van Duuren did, it is expected that the characterization can be further developed and verified.

ESTIMATION OF $\sigma'_y$

In buoyancy-dominated plumes, the plume cross section concentration pattern, although complex, is usually considered symmetrical with the lateral and vertical dimensions being equal; see, for example, the discussion on page 63 of Briggs (1969). Although allowances were recommended (Hanna et al.,1977) for buoyancyinduced dispersion in the lateral direction, the problem was not discussed explicitly. However, Briggs (personal conversations) suggests, for

for simple Gaussian plume modeling, it is appropriate to assume that the induced lateral dispersion due to buoyancy $\sigma'_y$ is equal to the induced vertical dispersion due to buoyancy $\sigma'_z$. Hence, $\sigma'_y$ can be estimated as:

$$\sigma'_y = \sigma'_z = \Delta h/3.5 \qquad (18)$$

ESTIMATION OF $\sigma''_y$

Pasquill (1976, 1974) discusses the effects of wind direction shear in the vertical on the time-averaged lateral dispersion. Interaction between the vertical spread and the turning of the wind direction with height enhances the lateral dispersion. Inspection of instantaneous cross sections of plumes, such as developed by aircraft sampling, reveals systematic displacement of the plumes in accordance with the veering of the wind direction with height; see, for example, Schiermeier and Niemeyer (1970) or Brown et al. (1972). However, time-averaged crosswind distributions at various levels do not reveal similar structure in accordance with the veering action of the wind direction; see discussion on page 229 of Pasquill (1974). Pasquill notes that a complete understanding of the importance of the induced dispersion due to wind direction shear in the vertical $\sigma''_y$ has not been reached. However, he suggests as a rough rule, that the total instantaneous crosswind spread (expressed in an angular sense) at large downwind distances, on the order of 20 to 100 km, is equal to 0.75 $\Delta\theta$, where $\Delta\theta$ is the change of wind direction over the entire vertical extent of the plume. Hence, $\sigma''_y$ can be estimated as:

$$\sigma''_y = (0.75/4.3) \times \Delta\theta = 0.174 \times \Delta\theta \qquad (19)$$

Here, x is the distance of transport downwind.

For estimates of the total time-averaged lateral dispersion $\sigma_y$, $\sigma''_y$ would have to be combined with the contribution to the lateral dispersion due to wind direction fluctuations $\sigma_{yo}$. Existing data suggest $\sigma''_y$ contributes little to the total lateral dispersion for downwind distances of less than 10 km. For downwind distances more than 20 km, the effect is more likely to be important in characterizing the total lateral dispersion.

## SECTION 4

## OVERVIEW

In Section 2 and Section 3, some of the techniques available for modeling various processes affecting dispersion from elevated releases were discussed. Based on the currently available data and results, a technique was selected for estimating each of the processes. Further development and refinement is needed in all of the techniques selected. Indeed, some of the techniques represent approximations developed more through a combination of theory and inferences than from analysis of field data designed specifically to investigate the particular dispersive process. The discussion presented in this section is devoted to an overview of the generalized scheme which can be formed through the synthesis of the selected techniques. Some of the difficulties of the various techniques selected have been noted in Sections 2 and 3. Other difficulties that arise from synthesizing the techniques into one scheme will be discussed in this section. Appendix B presents a FORTRAN listing that performs the computations outlined in the following discussion.

### VERTICAL DISPERSION

Based on the discussions within Sections 1 and 2, the total vertical dispersion, $\sigma_z$, can be approximated as:

$$\sigma_z^2 = (\sigma_w \; t \; F_z)^2 + (\Delta h/3.5)^2 \qquad (20)$$

where $\sigma_w$ = total standard deviation of the vertical wind component at the effective release height $H_e$

$t$ = travel time downwind, usually approximated as $x/u_{h_s}$ where x is the downwind distance and $u_{h_s}$ is the wind speed at the release height $h_s$

$\Delta h$ = buoyant plume rise

$H_e = h_s + \Delta h$, the effective release height

The dispersion function, $F_z$, is a function of stability, effective release height, and travel time downwind.

As discussed in Section 2, the parameters useful for characterizing $F_z$ are different for convectively unstable conditions than for neutral and stable conditions. During convective conditions, the convective velocity scale, $w_*$, is an appropriate scaling parameter; however, as conditions become neutral ($-L \rightarrow \infty$), $w_* \rightarrow 0$ (Equation 5). Furthermore, $w_*$ is appropriate for use only within the convectively mixed layer. This convective layer extends from a height $|L|$ from the surface to a height h from the surface. The layer extending from the surface to height $|L|$ is considered the surface layer, or constant stress layer, where $u_*$ (the surface friction velocity) is nearly invariant with height. The characterizations of $F_z$ presented in Figures 1 and 2 are for dispersion within the convective layer when conditions are convectively unstable (roughly, when $-h/L > 10$). Inspection of Figure 1 suggests that $F_z$ is independent of effective release height for scaled heights $H_e/h$ greater than 0.25. For releases where $H_e/h$ is less than 0.25, $F_z$ seems to be a function of height. A comparison of the results depicted in Figure 1 with those depicted in Figure 2, suggests that when $H_e/h = 0.067$, $F_z$ could be estimated using a linear interpolation between $F_z$ for $H_e/h$ of 0.25 and $F_z$ for $H_e/h$ of 0.025.

For releases where $H_e/h$ is less than 0.025, $F_z$ could be estimated by extrapolation. Care must be taken in such extrapolations that $H_e$ is not within the surface layer; in other words, $-H_e/L$ should be greater than 1. However, this restriction may not be sufficient. For instance, convectively unstable parameterizations of the vertical velocity fluctuations seem to fail for heights less than 0.005 h; see Figure 4 of Irwin (1979). Hence, the results presented in Figures 1 and 2 are considered useful for characterizing $F_z$ for effective release heights greater than 0.005 h and less than h when $-h/L$ is greater than 10.

The preceding paragraph discussed some of the limitations of the characterization presented for $F_z$ for unstable conditions. The following discussion addresses some of the limitations of the $F_z$ characterizations presented for neutral and stable conditions. During stable conditions, it is convenient to envision the atmosphere as being divided into a number of horizontal

layers. In the surface layer, extending to a height of order |L| the surface friction velocity is nearly invariant with height. Above this surface layer is the transitional layer (Planetary Boundary Layer) between the disturbed flow within the surface layer and the smooth frictionless flow of the free atmosphere. The characterization of $F_z$ presented for neutral and stable conditions is considered useful within the Planetary Boundary Layer and the surface layer. Of the dispersion data available, the majority is for near-neutral conditions. Hence, the characterization of $F_z$ is tentative, especially for stable conditions. Fortunately, the vertical dispersion due to atmospheric turbulence, $\sigma_{zo}$, is at a minimum during stable conditions. Hence, for elevated releases, the tentative nature of $F_z$ will be of little concern so long as the terrain is flat and conditions are steady-state. If conditions are not steady-state during the transport downwind or the terrain is not flat, none of the characterizations presented for $F_z$ are likely to be relevant.

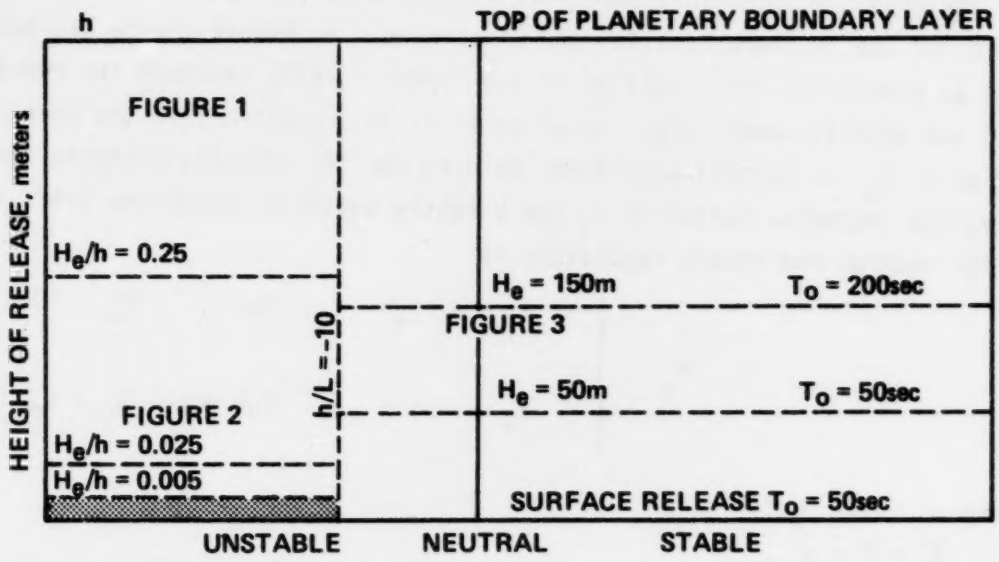


Figure 5. Summary diagram of the characterization of the vertical dispersion function, $F_z$, as a function of effective release height, $H_e$, and stability.

Figure 5 summarizes the preceding discussion on characterizing $F_z$. Based on the preceding discussion, $F_z$ for unstable conditions when -h/L is greater than 10:

$$F_z = \begin{cases} \alpha f_s + \beta f_e & \text{when } 0.005 < H_e/h < 0.25 \\ f_e & \text{when } H_e/h \geq 0.25 \end{cases} \quad (21)$$

where $\alpha = (10/9)(1 - 4H_e/h)$

$\beta = 1 - \alpha$

$$f_s = \begin{cases} \{(1+1.88\ t^*)/(1+\ t^{*3})\}^{1/2} - 4.5\ t^*(1-\ t^*)^4 & \text{for } t^* < 1 \\ (1.44/t^*)^{1/2} & \text{for } t^* \geq 1 \end{cases}$$

$$f_e = \begin{cases} 1 - 0.7\ t^* + 0.2\ t^{*2} & \text{for } t^* < 1 \\ (0.25/t^*)^{1/2} & \text{for } t^* \geq 1 \end{cases}$$

$t^* = w_* t/h$

For unstable conditions when -h/L is less than 10, $w_*$ is no longer appropriate for use in characterizing the dispersion. No simple device has been found to transition from Equation 21 and asymptotically approach the results valid for neutral conditions. At present, it is suggested that the characterization of $F_z$ for neutral conditions be used for the slightly unstable conditions. Hence, the characterization of $F_z$ for slightly unstable conditions (-h/L < 10) and for neutral and stable conditions is:

$$F_z = \begin{cases} \alpha' f_s' + \beta' f_e' & \text{for } H_e < 50 \text{ m} \\ f_e' & \text{for } 50\text{m} \leq H_e < h_{PBL} \end{cases} \quad (22)$$

where $\alpha' = 1 - H_e/50\text{m}$

$\beta' = 1 - \alpha'$

$h_{PBL}$ = the depth of the Planetary Boundary Layer

$f_s' = 1/\{1+ 0.9(t/T_o)^{1/2}\}$

$f_e' = 1/\{1+ 0.945(t/T_o)^{0.806}\}$

$$T_o(s) = \begin{cases} 50\ s & H_e \leq 50\ m \\ (3H_e - 50m)/2m & 50m < H_e < 150\ m \\ 200\ s & H_e \geq 150\ m \end{cases}$$

The gray hatched area in Figure 5 for $H_e/h$ less than 0.005 denotes the situations where the characterization of $F_z$ (Equations 21 and 22) has not been specified.

For nonbuoyant releases when $H_e/h$ is greater than 0.25, Lamb (1979) suggests that $H_e$ in the Gaussian plume model be adjusted in order to properly estimate surface concentrations. For instance, for an elevated nonbuoyant release, the Gaussian plume equation (for estimating maximum surface concentrations beneath the centerline of the plume, for downwind distances where the dispersion has not been affected by the presence of a limit to dispersion h) could be written as:

$$\chi = \frac{Q}{\pi \sigma_y \sigma_z u_{h_s}} \exp\left[-\frac{1}{2}\left(\frac{R H_e}{\sigma_z}\right)^2\right] \qquad (23)$$

where $\chi$ = the surface concentration, $g/m^3$

$Q$ = the emission rate, g/s

$H_e$ = the effective release height; in this case, simply $h_s$, if $\Delta h$ is zero

R is the correction factor to $H_e$ needed to force Equation 23 to yield proper estimates of the surface concentrations resulting from dispersion from a nonbuoyant elevated release. R varies as a function of downwind distance as depicted in Figure 6 of Lamb (1979). Simple approximations to Lamb's results are, for $H_e/h \geq 0.25$

$$R = \begin{cases} 1 & \text{for } x' \leq 1 \\ h/4 + (H_e - h/4)(2 - x') & \text{for } 1 < x' < 2 \\ h/4 & \text{for } x' \geq 2 \end{cases} \qquad (24)$$

where $x' = (x/H_e)(w_*/u_{h_s})$

For $H_e/h < 0.25$,

$$R = 1 \qquad (25)$$

Equation 24 and 25 simplify Lamb's results. This is especially true for Equation 25, since the locus of maximum concentration tends to rise off the surface for surface releases in convectively unstable conditions. However, the maximum surface concentration for surface releases occurs so close to the release point that R is essentially equal to 1. It is important to stress that Equations 24 and 25 have been shown necessary only for nonbuoyant releases when conditions are convectively unstable. Later studies may suggest that R needs further characterization to handle other situations which have a non-Gaussian distribution of pollutant material in the vertical or for certain types of buoyant releases. At present, R is set to 1 for all cases except those cases meeting the criteria set forth when Equation 24 is appropriate.

## LATERAL DISPERSION

Based on the discussions within Sections 1 and 3, the total lateral dispersion $\sigma_y$ can be approximated as;

$$\sigma_y^2 = (\sigma_v \; t \; F_y)^2 + (\Delta h/3.5)^2 + (0.174 \times \Delta\theta)^2 \qquad (26)$$

where $\sigma_v$ = the total standard deviation of the lateral wind component at the effective release height (should have an averaging time the same as the averaging time to be associated with the concentration estimate)

$\Delta\theta$ = the average change in the horizontal wind direction over the vertical extent of the plume (in radians)

The dispersion function $F_y$ is mainly a function of stability and travel time downwind.

As discussed in Section 3, the parameters useful for characterizing $F_y$ change in going from convectively unstable conditions to neutral and stable conditions. Whereas $w_*$ is the appropriate scaling velocity during convectively unstable conditions, $u_*$ is the appropriate scaling velocity within the surface layer (during stable or unstable conditions) and within the Planetary Boundary Layer during neutral and stable conditions. Hence, the characterization of $F_y$ presented in Section 3 is appropriate except for effective release heights less than 0.005 h when conditions are convectively unstable. Also, $T_o$ was chosen to

be 1000 seconds. (Draxler (1976) suggests $T_o$ is 300 s for surface releases during stable conditions). Hence, the characterization of $F_y$ presented is not suggested for releases less than 50 m.

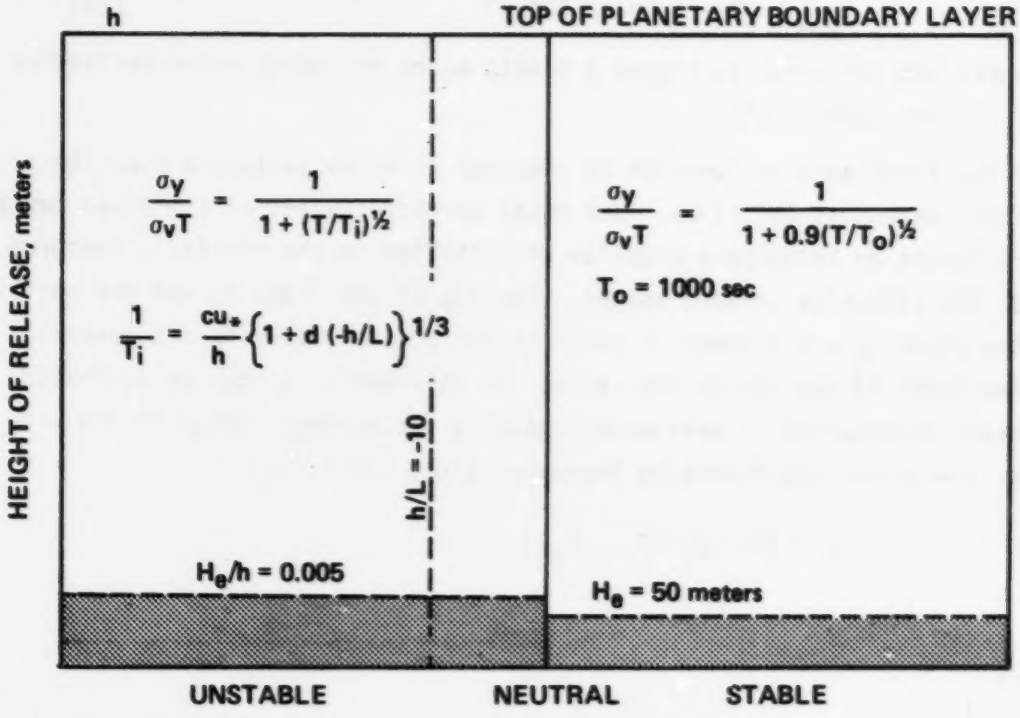


Figure 6. Summary diagram of the characterization of the lateral dispersion function, $F_y$, as a function of effective release height, $H_e$, and stability.

Figure 6 summarizes the preceding discussions regarding the characterization of $F_y$. Specfically, for unstable conditions when $H_e/h$ is greater than 0.005 (or $H_e$ is greater than 50 m - whichever is the larger height), $F_y$ can be approximated as:

$$F_y = 1/\{1 + (t/T_i)^{1/2}\} \qquad (27)$$

where $T_i^{-1} = (2.5\ u_*/h)\{1 + 0.00133(-h/L)\}^{1/3}$

The value of the von Karman constant assumed in the above expressions is 0.4. For neutral and stable conditions:

$$F_y = 1/\{1 + 0.90(t/1000s)^{\frac{1}{2}} \qquad (28)$$

The gray hatched areas in Figure 6 denote where the above characterization of $F_y$ is not appropriate.

The third term in Equation 26 requires $\Delta\theta$ to be evaluated over the vertical extent of the plume. The total vertical extent of the plume can be approximated by assuming a Gaussian distribution in the vertical, centered about the effective release height. The top of the plume $T_P$ and the bottom of the plume $T_B$ are assumed to occur at the points where the concentration is one-tenth of the centerline value. On this basis, $\Delta\theta$ can be estimated, provided information is available regarding the average change in the wind direction within the Planetary Boundary Layer, $\partial\theta/\partial z$, as:

$$\Delta\theta = (\partial\theta/\partial z)\ (T_P - T_B)$$

where

$$T_P = \begin{cases} \text{the lesser of } h \text{ or } H_e + 2.15\ \sigma_z & \text{for } L \leq 0 \\ H_e + 2.15\ \sigma_z & \text{for } L > 0 \end{cases}$$

$T_B$ = the greater of zero or $H_e - 2.15\ \sigma_z$

In most applications, this third term in Equation 26 can be neglected for downwind distances less than 10 km.

# SECTION 5
# CONCLUSION

The dispersion parameter estimation scheme presented was designed particularly for Gaussian plume modeling of elevated releases from tall sources. The scheme incorporates results from various studies of dispersion. Once the scheme is cast in a more practical form, it is anticipated to prove useful for characterizing dispersion in a variety of situations, even though the scheme is restricted to modeling applications having flat, homogeneous terrain and having steady-state meteorological conditions. Through the incorporation of dispersion results from elevated releases in unstable to slightly unstable conditons, the characterization of $F_z$ might be simplified, much as the characterization of $F_y$ was by Nieuwstadt and Van Duuren (1979) results. The characterizations of $F_z$ and $F_y$ might also be expanded to include surface releases; the scheme could then be used with no release height restrictions. This report documents the scheme in its most basic form. Further analysis is required before the input parameters can be fully discussed. The scheme in its basic form requires meteorological data not routinely available, such as the Monin-Obukhov length scale, the surface friction velocity, and the standard deviations of the vertical and lateral velocity components at the effective height of release.

In order to further develop the scheme for routine use, information is needed concerning the sensitivity of the scheme to the various input parameters. The sensitivity analysis will help to reveal

- the necessary precision and accuracy for specifying input parameters,
- input parameters to be simplified or eliminated.

Once the sensitivity analysis has been performed, then a field data set can be sought to evaluate the performance of the scheme for characterizing the dispersion from an elevated release. The results of the sensitivity analysis will be used to evaluate existing data bases and decide which data bases have the input specified sufficiently in order to properly evaluate the performance of the scheme.

For widespread use of the scheme (and perhaps even for the performance evaluation), some of the input, such as $u_*$, L, $\sigma_w$, $\sigma_v$, and h, most likely will have to be estimated from routinely available meteorological data. Here again, the results of the sensitivity analysis will prove useful. The characterizations selected to estimate the input parameters from routinely available data can be developed to satisfy accuracy and precision constraints specified by the sensitivity analysis.

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# APPENDIX A

## LIST OF DATA

The following discussion presents the dispersion data from various studies that were used to estimate the change in the vertical dispersion versus travel time. For each data set some assumptions were necessary, since at present no comprehensive data set exists that contains all the required information. The assumptions made are felt to be consistent with the intent of developing a generalized characterization of dispersion for use in the interim until a more complete scheme can be developed through the acquisition of new data.

Of the parameters required to characterize the vertical dispersion function $F_z = \sigma_z/(\sigma_w t)$, typically $\sigma_w$ (the standard deviation of the vertical wind component at the effective release height) is the one parameter that most often is not explicitly given. During convectively unstable conditions, the variation of $\sigma_w$ within the convective layer has been characterized by several authors using field data, numerical data, and tank data. These results were summarized in Figure 4 of Irwin (1979) and reproduced here as Figure A-1. This figure can be used to estimate $\sigma_w/w_*$ during convectively unstable conditions as a function of $H_e/h$, where $H_e$ is the effective release height and h is the depth of the convectively mixed layer. The semi-empirical relationships employing similarity scaling theory can be used to estimate $u_*$ (the surface friction velocity) and $\sigma_w$ (within the surface layer), given an estimate of $Z_o$ (the surface roughness length), L (the Monin-Obukhov scaling length), and a measurement of the horizontal wind speed at a given height within the surface layer. For neutral conditions where 1/L is zero, the nondimensional wind shear can be estimated as (Benoit, 1977):

$$u/u_* = (1/k)\ \ln(Z/Z_o) \qquad \text{(A-1)}$$

and $\sigma_w$ within the surface layer can be estimated as (Binkowski, 1979):

$$\sigma_w/u_* = 1.277 \qquad \text{(A-2)}$$

If the vertical dispersion $\sigma_z$ is given as a function of downwind distance, $F_z$ values can be estimated as:

$$F_z = \sigma_z/(\sigma_w t) \triangleq \sigma_z/(\sigma_e x) \qquad \text{(A-3)}$$

where the standard deviation of the vertical wind direction $\sigma_e$ could be estimated as:

$$\sigma_e \triangleq \sigma_w/u = 1.277\ k/\ln(Z/Z_o) \qquad \text{(A-4)}$$

using Equations A-1 and A-2.

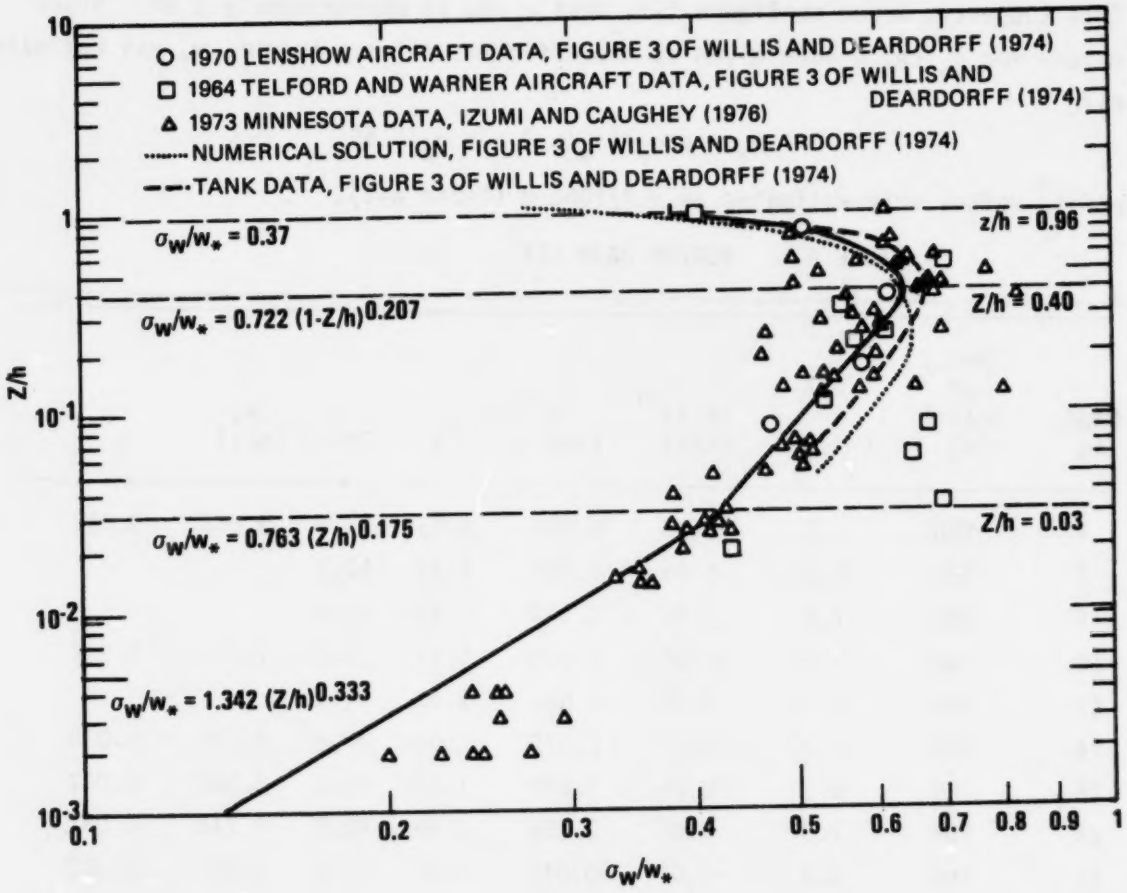


Figure A-1. Vertical profile of $\sigma_w/w_*$ for fully convective conditions. $\sigma_w$ is the standard deviation of the vertical velocity fluctuations, $w_*$ is the convective velocity scale, and h is the depth of the convectively mixed layer. Solution shown as solid line was derived, for Z/h greater than 0.03, from hand-drawn fits to the data values. Solid line, for Z/h less than 0.03, is prediction for free convection conditions (Equation 17 of Kaimal et al., 1976).

PORTON DATA SET (Hay and Pasquill, 1957)

In order to estimate parameters needed to characterize $F_z$ during unstable conditions, the depth of the convectively mixed layer h was estimated for the Porton data as 1000 m. For unstable conditions $H_e/h$ is then approximately 0.15. This suggests, based on Figure A-1, that $\sigma_w/w_*$ is approximately 0.55. Since values for $\sigma_e$ and u were given at the effective release height, $w_*$ was estimated as:

$$w_* \triangleq u\ \sigma_e/(\sigma_w/w_*) = u\ \sigma_e/0.55$$

and $t^*$ values were estimated as $w_* t/1000$ m (Table A-1).

TABLE A-1. PORTON DATA SET†

| Expt. no | Dist. of Travel (m) | $u^+$ (m/s) | $\Delta\theta/\Delta z^{++}$ (K/m) | $\sigma_z$ (rad.) | $F_z$ | t (sec) | $w_*$ (m/s) | $t^*$ |
|---|---|---|---|---|---|---|---|---|
| 4 | 100 | 6.7 | -0.05 | 0.054 | 0.55* | 14.9 | 0.658 | 0.010 |
| 7 | 100 | 8.0 | 0.03 | 0.052 | 1.23 | 12.5 | | |
| 8 | 100 | 7.4 | 0.01 | 0.049 | 1.25 | 13.5 | | |
| 12 | 300 | 7.7 | -0.03 | 0.080 | 1.07 | 39.0 | 1.12 | 0.044 |
| 13 | 300 | 7.3 | 0.07 | 0.066 | 1.03 | 41.1 | | |
| 14 | 300 | 6.3 | -0.03 | 0.070 | 1.03 | 47.6 | 0.802 | 0.038 |
| 15 | 300 | 6.1 | -0.05 | 0.086 | 1.00 | 49.2 | 0.954 | 0.047 |
| 16 | 500 | 7.3 | -0.06 | 0.056 | 0.94 | 68.5 | 0.743 | 0.051 |
| 17 | 500 | 6.8 | -0.02 | 0.047 | 1.07 | 73.5 | 0.581 | 0.043 |
| 19 | 300 | 9.5 | 0.03 | 0.070 | 1.50 | 31.6 | | |

† Data are from Hay and Pasquill (1957).

+ u equals the horizontal wind speed at the release height of 152.4 m.

++ The potential temperature lapse rate calculated using the reported temperature difference between the 23 and 4-ft levels.

* Experiment 4 is suggested by Hay and Pasquill as possibly being anomalous.

TANK DATA (Willis and Deardorff, 1976, 1978)

The results reported by Willis and Deardorff (1976, 1978) were for tank studies that modeled convectively unstable conditions. Using Figure 3 of Willis and Deardorff (1976), values of nondimensional vertical dispersion, $\sigma_z^* = \sigma_z/h$, were extracted as a function of nondimensional time, $t^* = w_* t/h$. Similarly, using Figure 2 of Willis and Deardorff (1978), values were extracted for $\sigma_z^*$ as a function of $t^*$. The results of Willis and Deardorff (1976) were for scaled release height, $H_e/h$, of 0.067, and, using the results of Figure A-1, $\sigma_w/w_*$ was estimated to be 0.50. The results of Willis and Deardorff (1978) were for a scaled release height of 0.24, and, using Figure A-1, $\sigma_w/w_*$ was estimated to be 0.63 (Table A-2).

It is important to realize that the values of $\sigma_z$, used to compute $\sigma_z^*$, were the actual standard deviation of the observed particles. The distribution of the particles may have been other than Gaussian. If particle distribution is not Gaussian, then use of these values of $\sigma_z^*$ may not accurately estimate surface concentrations if used unadjusted in a Gaussian dispersion model.

TABLE A-2. TANK DATA SET*

| $H_e/h = 0.067$ | | | $\sigma_w/w_* = 0.50$ | | |
|---|---|---|---|---|---|
| $t^*$ | $\sigma_z^*$ | $F_z$ | $t^*$ | $\sigma_z^*$ | $F_z$ |
| 0.13 | 0.05 | 0.77 | 0.82 | 0.44 | 1.07 |
| 0.18 | 0.07 | 0.78 | 0.88 | 0.46 | 1.05 |
| 0.25 | 0.11 | 0.88 | 0.95 | 0.48 | 1.01 |
| 0.37 | 0.19 | 1.03 | 1.02 | 0.50 | 0.98 |
| 0.44 | 0.23 | 1.05 | 1.08 | 0.52 | 0.97 |
| 0.50 | 0.26 | 1.04 | 1.24 | 0.56 | 0.90 |
| 0.55 | 0.29 | 1.06 | 1.89 | 0.57 | 0.60 |
| 0.64 | 0.33 | 1.03 | 2.23 | 0.55 | 0.49 |
| 0.70 | 0.37 | 1.06 | 2.61 | 0.53 | 0.40 |
| 0.78 | 0.41 | 1.06 | 2.89 | 0.53 | 0.37 |
| $H_e/h = 0.24$ | | | $\sigma_w/w_* = 0.63$ | | |
| 0.12 | 0.085 | 1.12 | 0.82 | 0.260 | 0.50 |
| 0.21 | 0.120 | 0.91 | 0.93 | 0.268 | 0.46 |
| 0.27 | 0.150 | 0.88 | 1.05 | 0.282 | 0.43 |
| 0.33 | 0.170 | 0.82 | 1.17 | 0.295 | 0.40 |
| 0.39 | 0.186 | 0.76 | 1.47 | 0.312 | 0.34 |
| 0.45 | 0.204 | 0.72 | 1.82 | 0.350 | 0.31 |
| 0.50 | 0.210 | 0.67 | 2.23 | 0.388 | 0.28 |
| 0.57 | 0.225 | 0.63 | 2.80 | 0.390 | 0.22 |
| 0.63 | 0.236 | 0.59 | 3.24 | 0.374 | 0.18 |
| 0.71 | 0.250 | 0.56 | | | |

* Data taken from Willis and Deardorff (1976, 1978).

NUMERICAL DATA SET (Lamb, 1978)

The results of Lamb (1978) were for a numerical simulation of convectively unstable conditions where the scaled release height, $H_e/h$, was 0.26. In conversations with Lamb, I found that studies for other release heights had been accomplished but were as yet unpublished. These unpublished results, in part, were used in this analysis and are listed (Table A-3). For each release height the scaled release height was used to estimate the scaled vertical velocity fluctuations from Figure A-1.

The values of $\sigma_z$, used to compute $\sigma_z^*$, were the actual standard deviations of the modeled particles. Use of these values of $\sigma_z^*$, without adjustment in a Gaussian dispersion model may not yield valid estimates of surface concentrations. My conversations with Lamb revealed that the distributions of the particle were not Gaussian for the elevated releases. The distributions were skewed with more particles towards the surface; hence, use of these $\sigma_z^*$ values in a Gaussian dispersion model (without adjustment) tends to underestimate surface concentrations. The skewed distributions were noticed by Lamb for his simulations with nonbuoyant releases. However, the distributions for buoyant releases appear to be more Gaussian. Lamb has only begun to investigate dispersion from buoyant releases. Hence, the latter result reported is of a very preliminary nature.

TABLE A-3. NUMERICAL DATA SET[a]

| $H_e/h = 0.025$ | | | $\sigma_w/w_* = 0.392$ | | |
|---|---|---|---|---|---|
| $t^*$ | $\sigma_z^*$ | $F_z$ | $t^*$ | $\sigma_z^*$ | $F_z$ |
| 0.013 | 0.0047 | 0.922 | 0.717 | 0.3439 | 1.224 |
| 0.026 | 0.0090 | 0.883 | 0.810 | 0.3953 | 1.245 |
| 0.040 | 0.0130 | 0.829 | 0.902 | 0.4391 | 1.242 |
| 0.053 | 0.0167 | 0.804 | 1.008 | 0.4806 | 1.216 |
| 0.066 | 0.0203 | 0.785 | 1.100 | 0.5118 | 1.187 |
| 0.079 | 0.0240 | 0.775 | 1.281 | 0.5583 | 1.112 |
| 0.106 | 0.0321 | 0.773 | 1.323 | 0.5670 | 1.093 |
| 0.132 | 0.0409 | 0.790 | 1.442 | 0.5875 | 1.039 |
| 0.198 | 0.0666 | 0.858 | 1.766 | 0.6198 | 0.895 |
| 0.251 | 0.0905 | 0.920 | 1.951 | 0.6239 | 0.816 |
| 0.304 | 0.1160 | 0.973 | 2.083 | 0.6245 | 0.765 |
| 0.410 | 0.1704 | 1.060 | 2.232 | 0.6235 | 0.713 |
| 0.506 | 0.2219 | 1.119 | | | |
| 0.611 | 0.2822 | 1.178 | | | |
| $H_e/h = 0.26$ | | | $\sigma_w/w_* = 0.63$ | | |
| 0.106 | 0.0614 | 0.92 | 0.743 | 0.2758 | 0.59 |
| 0.132 | 0.0762 | 0.92 | 0.796 | 0.2824 | 0.56 |
| 0.202 | 0.1131 | 0.89 | 0.927 | 0.2974 | 0.51 |
| 0.268 | 0.1449 | 0.86 | 1.059 | 0.3128 | 0.47 |
| 0.347 | 0.1792 | 0.82 | 1.178 | 0.3269 | 0.44 |
| 0.387 | 0.1948 | 0.80 | 1.462 | 0.3638 | 0.39 |
| 0.439 | 0.2130 | 0.77 | 1.806 | 0.4006 | 0.35 |
| 0.506 | 0.2353 | 0.74 | 2.201 | 0.4183 | 0.30 |
| 0.558 | 0.2444 | 0.70 | 2.746 | 0.4277 | 0.25 |
| 0.625 | 0.2575 | 0.65 | 2.905 | 0.4310 | 0.24 |
| 0.677 | 0.2663 | 0.62 | | | |

TABLE A-3. (continued)

| $H_e/h = 0.50$ | | | $\sigma_w/w_* = 0.63$ | | |
|---|---|---|---|---|---|
| $t^*$ | $\sigma_z^*$ | $F_z$ | $t^*$ | $\sigma_z^*$ | $F_z$ |
| 0.053 | 0.0310 | 0.93 | 0.598 | 0.2568 | 0.68 |
| 0.066 | 0.0387 | 0.93 | 0.651 | 0.2694 | 0.65 |
| 0.079 | 0.0463 | 0.93 | 0.704 | 0.2807 | 0.63 |
| 0.093 | 0.0537 | 0.92 | 0.757 | 0.2905 | 0.61 |
| 0.106 | 0.0610 | 0.91 | 0.796 | 0.2970 | 0.59 |
| 0.119 | 0.0682 | 0.91 | 0.849 | 0.3042 | 0.57 |
| 0.132 | 0.0753 | 0.90 | 0.902 | 0.3096 | 0.54 |
| 0.162 | 0.0908 | 0.89 | 1.006 | 0.3146 | 0.49 |
| 0.202 | 0.1106 | 0.87 | 1.125 | 0.3180 | 0.45 |
| 0.254 | 0.1349 | 0.84 | 1.234 | 0.3164 | 0.41 |
| 0.307 | 0.1571 | 0.81 | 1.327 | 0.3148 | 0.38 |
| 0.347 | 0.1731 | 0.79 | 1.449 | 0.3123 | 0.34 |
| 0.400 | 0.1939 | 0.77 | 1.555 | 0.3093 | 0.31 |
| 0.453 | 0.2134 | 0.74 | 2.069 | 0.3086 | 0.24 |
| 0.506 | 0.2311 | 0.72 | 2.373 | 0.3107 | 0.21 |
| 0.545 | 0.2428 | 0.70 | | | |

TABLE A-3. (continued)

| $H_e/h = 0.50$ | | | $\sigma_w/w_* = 0.55$ | | |
|---|---|---|---|---|---|
| $t^*$ | $\sigma_z^*$ | $F_z$ | $t^*$ | $\sigma_z^*$ | $F_z$ |
| 0.013 | 0.0075 | 1.05 | 0.357 | 0.1354 | 0.69 |
| 0.026 | 0.0146 | 1.02 | 0.423 | 0.1531 | 0.66 |
| 0.040 | 0.0214 | 0.97 | 0.506 | 0.1762 | 0.63 |
| 0.053 | 0.0282 | 0.97 | 0.545 | 0.1873 | 0.62 |
| 0.066 | 0.0349 | 0.96 | 0.625 | 0.2097 | 0.61 |
| 0.079 | 0.0412 | 0.95 | 0.704 | 0.2329 | 0.60 |
| 0.093 | 0.0475 | 0.93 | 0.757 | 0.2491 | 0.60 |
| 0.106 | 0.0533 | 0.91 | 0.849 | 0.2772 | 0.59 |
| 0.119 | 0.0591 | 0.90 | 0.902 | 0.2937 | 0.59 |
| 0.132 | 0.0646 | 0.89 | 1.008 | 0.3246 | 0.59 |
| 0.198 | 0.0894 | 0.82 | 1.508 | 0.4342 | 0.52 |
| 0.251 | 0.1064 | 0.77 | 1.819 | 0.4540 | 0.45 |
| 0.304 | 0.1213 | 0.73 | 2.004 | 0.4502 | 0.41 |

[a] Data are from Lamb (1978).

HANFORD DATA SET (Hilst and Simpson, 1958)

Using the field data reported by Hilst and Simpson (1958), tests C and D were determined to be closest to neutral conditions. The data did not report the values of the standard deviation of the vertical velocity fluctuations, but data was given for the horizontal wind speed, u, at 60.96 m. Using Högström's (1964) estimate of the Hanford surface roughness length, $z_o$, of 0.05 m, values of the surface friction velocity $u_*$ were estimated using the reported wind speeds and the assumption that the stability was neutral. For test C, $u_*$ was estimated as 0.295 m/s; by means of Equation A-1 and the wind speed at the tracer release height of 56.4 was estimated to be 5.17 m/s. Using the estimated $u_*$ and Equation A-2, $\sigma_w$ was estimated to be 0.37 m/s. A similar analysis was performed on test D, yielding estimates with u and $\sigma_w$ at 56.4 m of 4.57 m/s and 0.33 m/s, respectively (Table A-4).

TABLE A-4. HANFORD DATA SET[a]

| Travel distance (m) | Travel time (sec) | $\sigma_z$ (m) | $F_z$ |
|---|---|---|---|
| | Field test C | | |
| $H_e$ = 56.40 m | $Z_o$ = 0.05 m | u(60.96 m) = 5.23 m/s | |
| $u_*$ = 0.29 m/s | $\sigma_w$(56.4 m) = 0.37 m/s | u(56.40 m) = 5.17 m/s | |
| 152.4 | 29.9 | 5.5 | 0.50 |
| 304.8 | 59.8 | 8.9 | 0.40 |
| 762.0 | 149.4 | 13.6 | 0.25 |
| 1524.0 | 298.8 | 15.7 | 0.14 |
| | Field test D | | |
| $H_e$ = 56.40 m | $Z_o$ = 0.05 m | u(60.96 m) = 4.60 m/s | |
| $u_*$ = 0.26 m/s | $\sigma_w$(56.4 m) = 0.33 m/s | u(56.40 m) = 4.57 m/s | |
| 152.4 | 33.4 | 5.5 | 0.50 |
| 304.8 | 66.7 | 8.1 | 0.37 |
| 762.0 | 166.7 | 12.3 | 0.22 |
| 1542.0 | 333.5 | 14.8 | 0.13 |

[a] Data are from Hilst and Simpson (1958).

## STUDSVIK AND ÅGESTA DATA SET (Högström 1964)

Table 2 of Högström (1964) reports the dispersion results for Studsvik. The release height was 87 m for these data. The results for $\lambda$ equal to 0.3 in the table are the closest to near-neutral conditions. Högström (p. 221) suggests that the roughness length at Studsvik was 0.6 m and that $\sigma_w/u$ was 0.079 at 87 m. Using the reported value for $\sigma_w/u$, Equation A-3 would suggest that $Z_o$ equals about 0.10 m at Studsvik. The discrepancy between Högström's reported value for the surface roughness and the estimated value is evidence that the flow at Studsvik was not ideal. Högström noticed that terrain-induced anomolies existed in the turbulence profiles and wind speed profiles at Studsvik. For this study, $Z_o$ is assumed to be 0.1 m. If the flow was not steady-state, assuming $Z_o$ equals 0.1 m allows the Studsvik data to be used for the shorter travel times. Hence, the resulting estimates of $F_z$ for the longer travel times are considered suspect. The Agesta site had more ideal flow; hence, the Agesta wind information has been used to estimate the wind speed at Studsvik. Högström (p 226) suggests that $u(122m)/u_f$ = 0.71 where $u_f$ = 10 m/s. Using Equation A-1 and this estimate of wind speed at 122 m, $u_*$ is estimated to be 0.4 m/s, and u(87m) is estimated to be 6.8 m/s. With this estimate of u at 87 m and the reported value for $\sigma_w/u$, $\sigma_w$ is estimated to be 0.54 m/s (Table A-5).

Table 3 of Högström (1964) reports the results of dispersion studies at Ågesta. The release height for these data was 50 m. Unlike the Studsvik data, the Agestasite apparently presented no major terrain problems. For the neutral data ($\lambda = -\infty$), Högström reports $\sigma_w/u$ at 50 m as 0.12. Using Equation A-3, this suggests $Z_o$ is 0.59 m, which agrees with Högström's estimate of $Z_o$. Assuming u(122m) is 7.1 m/s and $Z_o$ is 0.59 m, Equation A-1 estimates $u_*$ to be 0.53 m/s and u(50m) to be 5.9 m/s. With the given value for $\sigma_w/u$ and this estimate of u(50m), $\sigma_w$ is estimated to be 0.71 m/s (Table A-5).

TABLE A-5. STUDSVIK AND ÅGESTA DATA SET*

| Studsvik - 87 m ($\lambda$ = 0.3) | | | | Ågesta - 50 m ($\lambda = -\infty$) | | | |
|---|---|---|---|---|---|---|---|
| Travel distance (m) | $\sigma_z$ (m) | Travel time (s) | $F_z$ | Travel distance (m) | $\sigma_z$ (m) | Travel time (s) | $F_z$ |
| 100 | 7.0 | 14.7 | 0.89 | 50 | 5.0 | 8.5 | 0.83 |
| 250 | 15.0 | 36.8 | 0.76 | 100 | 8.3 | 17.0 | 0.69 |
| 500 | 24.7 | 73.5 | 0.63 | 150 | 12.2 | 25.4 | 0.68 |
| 1000 | 38.3 | 147.1 | 0.49 | 250 | 17.8 | 42.4 | 0.59 |
| 2000 | 57.6 | 294.1 | 0.37 | 500 | 31.6 | 84.8 | 0.53 |
| 3000 | 69.5 | 441.2 | 0.29 | 750 | 38.5 | 127.1 | 0.43 |
| 4000 | 80.5 | 588.2 | 0.26 | 1000 | 46.2 | 169.5 | 0.39 |
| 5000 | 88.0 | 735.3 | 0.22 | | | | |

* Data are from Högström (1964).

SECOND ORDER CLOSURE DATA SET (Lewellen and Teske, 1975)

Figure 22 of Lewellen and Teske (1975) depicts the variation of vertical dispersion as a function of downwind distance for a surface release and an elevated release during neutral conditions. The plot has been normalized using scaling parameters. Figure 11 of their report suggests that results for neutral conditions can be interpreted dimensionally using:

$u_g$ = the geostrophic wind speed, 10 m/s

$Z_o$ = the surface roughness length, 10 cm

$f$ = the Coriolis parameter, $10^{-4} s^{-1}$

$R_o$ = the Rossby number, $u_g/(Z_o f)$, $10^6$

A typical wind speed at 10 m during neutral conditioons is 5 m/s. The velocity scale $\hat{u}$ as defined by Lewellen and Teske would then be 0.5 m/s and $\hat{R}_o = \hat{u}/(Z_o f)$ would be 50,000.

Figure 10 of their report suggests that for a surface release during neutral conditions $\sigma_w/\hat{u} = 1.1$; hence, $\sigma_w$ equals approximately 0.55 m/s. For the surface release, the transport wind speed was assumed to be 5 m/s.

For the elevated release ($H_e f/\hat{u} = 0.144$), $H_e$ is 570 m if $\hat{u}$ equals 0.5 m/s. Using Figure 7 of their report ($H_e f/u_g = 0.00057$), $(\sigma_w/u_g)^2$ is estimated to be 0.0011 and $u(570m)/u_g$ is estimated to be 0.96. Hence, $\sigma_w$ is estimated to be 0.332 m/s, and the transport wind speed is estimated to be 9.6 m/s.

With the above information, data values were extracted from Figure 22 of Lewellen and Teske's (1975) report and expressed in dimensional form (Table A-6).

TABLE A-6. SECOND ORDER CLOSURE DATA SET*

| $H_e$ = 0 | u = 5 m/s | $Z_o$ = 0.1 m | $\sigma_w$ = 0.55 m/s |
|---|---|---|---|
| Travel distance (km) | $\sigma_z$ (m) | Travel time (sec) | $F_z$ |
| 0.10 | 5.60 | 20 | 0.51 |
| 0.25 | 11.75 | 50 | 0.43 |
| 0.50 | 21.25 | 100 | 0.39 |
| 1.00 | 36.25 | 200 | 0.33 |
| 2.50 | 75.00 | 500 | 0.27 |
| 5.00 | 132.50 | 1000 | 0.24 |
| 10.00 | 212.50 | 2000 | 0.19 |
| 25.00 | 375.00 | 5000 | 0.14 |
| 50.00 | 550.00 | 10000 | 0.10 |
| $H_e$ = 570 m | u = 9.6 m/s | $Z_o$ = 0.1 m | $\sigma_w$ = 0.33 m/s |
| 0.14 | 5.00 | 14.6 | 1.02 |
| 0.25 | 7.60 | 26.0 | 0.87 |
| 0.50 | 12.75 | 52.1 | 0.73 |
| 1.00 | 21.50 | 104.2 | 0.61 |
| 2.50 | 41.50 | 260.4 | 0.47 |
| 5.00 | 71.00 | 520.8 | 0.41 |
| 10.00 | 110.00 | 1041.7 | 0.31 |
| 25.00 | 200.00 | 2604.2 | 0.23 |
| 50.00 | 280.00 | 5208.3 | 0.16 |

* Data are from Lewellen and Teske (1975).

## APPENDIX B

## SUBROUTINE SZSY

The following FORTRAN subroutine performs the computations, as outlined in Section 4, for estimating the total vertical and lateral dispersion from an elevated point source. The subroutine was constructed in preparation for the further development and evaluations needed.

```
      SUBROUTINE SZSY(XKM,OLL,HS,DH,U,HL,DTHETA,USTAR,SE,SA,FSZ,FSY,
     CSZ,SY,RZR,IKEY,XTI,XXO)
C
C     THIS SUBROUTINE CALCULATES THE VERTICAL AND HORIZONTAL DISPERSION
C     (SZ AND SY, RESPECTIVELY) IN METERS.  ALL UNITS OF INPUT PARA-
C     -METERS ARE IN METERS, SECONDS, AND RADIANS EXCEPT FOR XKM WHICH
C     IS IN KILOMETERS.
C
C          THIS SUBROUTINE INCLUDES UPDATES AS OF 14 MARCH 1979.
C         PROGRAMMER:  JOHN S. IRWIN
C                      EPA (MD 80)
C                      RTP, NC 27711
C                      PHONE NUMBERS: COM. (919) 541-4564
C                                     FTS        629-4564
C
C
C                     GLOSSARY OF TERMS
C
C          INPUT PARAMETERS:
C
C     XKM = DOWNWIND DISTANCE IN KILOMETERS;
C     OLL = 1/L, WHERE L IS THE MONIN-OBUKHOV SCALING LENGTH
C           IN METERS.  THIS ROUTINE ASSUMES THAT THE VON
C           KARMAN CONSTANT IS EQUAL TO 0.4.
C      HS = STACK HEIGHT IN METERS.
C      DH = THE PLUME RISE IN METERS, (NOTE DH MUST BE GREATER THAN OR
C           EQUAL TO ZERO).
C       U = TRANSPORT WIND SPEED IN METERS PER SECOND, (NOTE MUST BE
C           GREATER THAN ZERO).
C      HL = IF L IS LE. ZERO, HL IS THE DEPTH OF THE CONVECTIVELY
C           MIXED LAYER IN METERS.  IF L IS GT. ZERO, HL IS THE
C           DEPTH OF THE STABLE SURFACE LAYER IN METERS.
C  DTHETA = THE RATE OF TURNING OF THE HORIZONTAL WIND DIRECTION IN
C           RADIANS PER METER.
C   USTAR = THE SURFACE FRICTION VELOCITY SCALE IN METERS PER SECOND.
C           FOR UNSTABLE CONDITIONS WE USE WSTAR INSTEAD OF USTAR.
C           WSTAR = USTAR*(-HL/(0.4*L) )**0.33333
C      SE = THE STANDARD DEVIATION OF THE VERTICAL WIND DIRECTION IN
C           RADIANS.
C      SA = THE STANDARD DEVIATION OF THE HORIZONTAL WIND DIRECTION IN
C           RADIANS.
C
C                     OUTPUT PARAMETERS
C
C     FSZ = SZ / ( SE*X) IN RADIANS** -1.
C     FSY = SY / ( SA*X) IN RADIANS** -1.
C      SZ = THE VERTICAL DISPERSION PARAMETER IN METERS.
C      SY = THE HORIZONTAL DISPERSION PARAMETER IN METERS.
C     RZR = THE FRACTIONAL ADJUSTMENT TO BE APPLIED TO THE EFFECTIVE
C           STACK HEIGHT IN THE GAUSSIAN PLUME DISPERSION MODEL.
```

```
C           RZR IS ALWAYS 1.0 UNLESS PLUME IS NONBUOYANT AND
C           MEETS SOME OTHER CONDITIONS, SEE PARA. 3 IN SECTION OF
C           OF COMMENTS ENTITLED 'INTERPOLATION SCHEMES'.
C     IKEY = IF NO ERRORS ARE DETECTED IN THE INPUT DATA, IKEY IS +1.
C           IF ERRORS ARE DETECTED IN THE INPUT DATA, EXECUTION IS
C           STOPPED AND IKEY IS SET TO -1.
C      XTI = U*TI, SEE PARA. 7 IN SECTION OF COMMENTS ENTITLED
C           'EQUATIONS OF INTEREST' FOR DEFINITION OF TI.
C      XTO = U*TO, SEE PARA. 7 IN SECTION OF COMMENTS ENTITLED
C           'EQUATIONS OF INTEREST' FOR DEFINITION OF TO.
C
C                       MATHEMATICAL MODEL
C
C      SZ = ( SZO**2 + SZ1**2 )**0.5
C           WHERE SZO = THE DISPERSION DUE TO TURBULENCE AND TRANSPORT.
C                 SZ1 = THE INDUCED DISPERSION DUE TO BUOYANT PLUME RISE,
C                       DH/3.5 .
C
C      SY = ( SYO**2 + SY1**2 + SY2**2 )**0.5
C           WHERE SYO = THE DISPERSION DUE TO TURBULENCE AND TRANSPORT,
C                 SY1 = THE INDUCED DISPERSION DUE TO BUOYANT PLUME RISE,
C                       DH/3.5 .
C                 SY2 = THE INDUCED DISPERSION DUE TO HORIZONTAL WIND
C                       DIRECTION SHEAR IN THE VERTICAL,
C                       0.173*DTHETA*(H1 - H2)*X
C                       AND H1 IS THE UPPER EXTENT OF THE PLUME AND H2
C                       IS THE LOWER EXTENT OF THE PLUME.  THE UPPER AND
C                       LOWER EXTENT OF THE PLUME ARE DEFINED AS THE
C                       EFFECTIVE PLUME HEIGHT, ZR=HS+DH, PLUS AND MINUS
C                       2.15SZ, UNLESS SUCH WOULD CAUSE H1 TO BE ABOVE
C                       THE CONVECTIVE MIXED LAYER LID HEIGHT OR H2 TO BE
C                       LESS THAN THE GROUND HEIGHT.
C
C                       INTERNAL PARAMETERS
C
C       X = XKM*1000, DOWNWIND DISTANCE IN METERS,
C      ZR = HS+DH, EFFECTIVE HEIGHT OF THE PLUME IN METERS,
C           NOTE, ZR IS SET TO 10.0 METERS IF IT IS LESS THAN 10 METERS.
C     SEE = SE, UNLESS SE IS LESS THAN 0.01.  IN SUCH CASES, SEE IS SET
C           TO 0.01 RADIANS.
C
C           SPECIFIC TO UNSTABLE ANALYSES
C
C  TSTAR = (X/U)*(WSTAR/HL), SCALED TRANSPORT TIME,
C  ZROHL = ZR/HL, SCALED EFFECTIVE RELEASE HEIGHT,
C    RZR = A FRACTION WHICH IS 1.0 UNLESS ZROHL IS GREATER THAN 0.25.
C           RZR IS THE FRACTIONAL ADJUSTMENT TO THE EFFECTIVE RELEASE
C           HEIGHT SUGGESTED BY R. LAMB'S NUMMERICAL MODEL RESULTS.
C
```

```
C          SPECIFIC TO NEUTRAL/STABLE ANALYSES
C
C     XO = U*OTO,  ANALOGOUS TO DRAXLER'S TO EXCEPT HERE WE ARE
C          IN METERS.  TO AND XO ARE EMPIRICAL PARAMETERS RELATED
C          TO THE LAGRANGIAN TIME SCALE.
C    OTO = AN EMPIRICAL PARAMETER RELATED TO THE LAGRANGIAN TIME SCALE.
C          IT IS ASSIGNED A VALUE BASED ON THE EFFECTIVE RELEASE
C          HEIGHT.  WHERE THE EFFECTIVE RELEASE HEIGHT (ZR)
C          IS EXPRESSED IN METERS AND OTO IS EVALUATED AS:
C          50 SEC IF ZR IS LESS THAN 50 METERS,
C          200 SEC IF ZR IS GREATER THAN 150 METERS, AND
C          (3*ZR - 50)/2 SEC FOR ZR VALUES INBETWEEN.
C
C                    EQUATIONS OF INTEREST
C
C     1. INTERPOLATION CONSTANTS USED FOR UNSTABLE CONDITIONS WHEN
C          ZROHL IS LESS THAN 0.25.
C     AU = (10/9)*(1 - 4ZR/HL)
C     BU =  1 - AU
C
C     2. INTERPOLATION CONSTANTS USED FOR NEUTRAL/STABLE ANALYSES WHEN
C          ZR IS LESS THAN 50 METERS.
C     AS = 1 - BS
C     BS = ZR/50.0
C
C     3. EQUATION " FSP "
C     IF TSTAR LE. 1.0, THEN
C     FSP = SQRT( (1.0+1.88*TSTAR)/(1.0+TSTAR**3) )
C                - 4.5*TSTAR*(1 - TSTAR)**4.0
C     IF TSTAR GT. 1.0, THEN
C     FSP = SQRT( 1.44/TSTAR )
C
C     4. EQUATION " FEP "
C     IF TSTAR LE. 1.0, THEN
C     FEP = 1 - 0.7*TSTAR + 0.2*TSTAR**2
C     IF TSTAR GT. 1.0, THEN
C     FEP = SQRT( 1/(4*TSTAR) )
C
C     5. EQUATION " FS "
C     FS = 1/( 1 + 0.9*SQRT(X/XO) )
C
C     6. EQUATION " FE "
C     FE = 1/( 1 + 0.945*(X/XO)**0.806 )
C
C     7. EQUATION FOR FSY (APPROXIMATION TO NIEUSTADT AND DUUREN,1979 RESULTS)
C          IF UNSTABLE:
C     OTI = (C*USTAR/HL)*(1+D(-HL/L))**1/3
C     WHERE C = 2.5
C           D = 0.00133
```

```
C     FSY = 1/(1+SQRT(T/TI))
C       NOTE TI = 1/OTI
C         IF NEUTRAL OR STABLE (DRAXLER'S RESULTS FOR ELEVATED RELEASES):
C     TTO = 1000 SEC
C     XXO = U*TTO, WHERE U IS IN M/S.
C     FSY = 1/(1+0.9*SQRT(X/XXO))
C
C                    INTERPOLATION SCHEMES
C
C     1. IF CONDITIONS ARE UNSTABLE AND ZROHL IS LT. 0.25, THEN
C
C     FSZ = AU*FSP + BU*FEP
C
C          OTHERWISE,
C
C     FSZ = FEP
C
C     2. IF CONDITIONS ARE NEUTRAL/STABLE AND ZR IS LT. 50 METERS, THEN
C
C     FSZ = AS*FS + BS*FE
C
C          OTHERWISE,
C
C     FSZ = FE
C
C     3. APPROXIMATION TO LAMB'S ADJUSTMENT TO EFFECTIVE RELEASE HEIGHT
C          WHEN CONDITIONS ARE UNSTABLE AND ZROHL IS GT. 0.25 .
C
C     IF TSTAR/ZROHL IS LT. 1.0, RZR = 1.0.
C     IF TSTAR/ZROHL IS GT 2.0, RZR = 0.25/ZROHL.
C     ALL OTHER CASES, RZR = 0.25/ZROHL +
C                    ( (ZROHL-0.25)/ZROHL )*( 2 - TSTAR/ZROHL ).
C
      IKEY = 1
      IF(DH.LT.0.0) GO TO 900
      IF(U.LE.0.0) GO TO 900
      X = XKM*1000.0
      ZR = HS + DH
      IF(ZR.LE.10.0) ZR = 10.0
      OL = OLL
      TEST = HL*OL
C
C     WE USE SEE FOR SE (INPUT) AND OL FOR OLL (INPUT)
C     BECAUSE WE MODIFY SEE AND OL IN THE
C     FOLLOWING ANALYSIS AND DO NOT WISH TO
C     CHANGE THE USER INPUT TO THE SUBROUTINE.
C
```

```
C*************************************************************************
C                                                                        *
C                                                                        *
C              START VERTICAL DISPERSION                                 *
C                 CALCULATIONS                                           *
C                                                                        *
C                                                                        *
C*************************************************************************
C
C     THE UNSTABLE ANALYSIS FOR DISPERSION IN THIS
C     ROUTINE IS KNOWN TO BE VALID FOR UNSTABLE
C     CONDITIONS WHERE ABS(HL/L) IS GREATER THAN OR EQUAL
C     TO 10.0.  WHEN ABS(HL/L) IS LESS THAN 10.0
C     AND L IS NEGATIVE (IE. UNSTABLE CONDITIONS)
C     IT MAY BE BETTER TO TREAT THE DISPERSION
C     AS IF IT WHERE NEUTRAL CONDITIONS BUT USE
C     THE TURBULENCE INTENSITIES APPROPRIATE
C     FOR THE STABILITY CONDITIONS (IE. THE ACTUAL
C     L IS USED TO DETERMINE SE AND SA).
C
C
      SEE = SE
      IF(SEE.LT.0.01) SEE = 0.01
      IF(ZR.LT.HL) GO TO 10
C
C     WE HAVE A CASE WHEN THE EFFECTIVE RELEASE
C     IS ABOVE THE SURFACE MIXED LAYER.  WE WILL
C     MODEL THIS AS IF CONDITIONS WERE NEUTRAL.
C
      OL = 0.0
      SEE = 0.01
      TEST = 0.0
   10 RZR = 1.0
C
C     TEST FOR STABILITY REGIME
C
      IF(TEST.LT.-10.0) GO TO 100
C
C***  MUST BE STABLE
C
      XO = 0.0
      OTO = 50.0
      IF(ZR.GT.50.0) OTO = (3.0*ZR-50.0)/2.0
      IF(ZR.GT.150.0) OTO = 200.0
      XO = U*OTO
      FE = 1.0/(1.0+0.945*(X/XO)**0.806)
      IF(ZR.LT.50.0) GO TO 50
      FSZ = FE
      GO TO 200
```

```
   50 CONTINUE
      FS = 1.0/(1.0+0.90*SQRT(X/XO))
      BS = ZR/50.0
      AS = 1 - BS
      FSZ = AS*FS + BS*FE
   75 GO TO 200
  100 CONTINUE
C
C***  MUST BE UNSTABLE CONDITIONS
C
      IF(HL.LE.0.0) GO TO 900
      IF(USTAR.LE.0.0) GO TO 900
      WSTAR = USTAR*(-HL*OL/0.4)**0.33333
      TSTAR=(X/U)*(WSTAR/HL)
      ZROHL = ZR/HL
      IF(ZROHL.GE.1.0) GO TO 900
      IF(TSTAR.LT.1.0) GO TO 120
      FEP= SQRT(1.0/(4.0*TSTAR))
      GO TO 125
  120 FEP= 1.0-0.70*TSTAR+0.20*TSTAR*TSTAR
  125 IF(ZROHL.LT.0.25) GO TO 145
      FSZ = FEP
      IF(DH.GT.0.0) GO TO 200
C
C     NOTICE WE APPLY THE CORRECTIONS TO THE
C     SCALING FACTOR FOR THE EFFECTIVE RELEASE
C     HEIGHT (RZR) AS LAMB SUGGEST ONLY WHEN
C     THE PLUME IS NONBUOYANT.
C
      XTSTAR = TSTAR/ZROHL
      IF(XTSTAR.LE.1.0) GO TO 200
      IF(XTSTAR.LT.2.0) GO TO 135
      RZR = 0.25/ZROHL
      GO TO 200
  135 RZR = (0.25/ZROHL) + ((ZROHL-0.25)/ZROHL)*(2.0-XTSTAR)
      GO TO 200
  145 IF(TSTAR.LT.1.0) GO TO 150
       FSP=SQRT(1.44/TSTAR)
      GO TO 160
  150 FSP = SQRT((1.0+1.88*TSTAR)/(1.0+TSTAR**3.0))
     C       - 4.5*TSTAR*(1.0-TSTAR)**4.0
  160 AU = (10.0/9.0)*(1.0 - 4.0*ZROHL)
      BU = 1.0 - AU
      FSZ = AU*FSP+BU*FEP
  200 SZO = SEE*X*FSZ
C     PLUME RISE EFFECTS
      SZ1 = DH/3.5
      SZ = SQRT(SZO*SZO+SZ1*SZ1)
      SY1 = SZ1
```

```
C
C*****************************************************************
C                                                                *
C                                                                *
C          FINISH VERTICAL DISPERSION                            *
C       CALCULATIONS AND START LATERAL DISPERSION                *
C                 CALCULATIONS                                   *
C                                                                *
C                                                                *
C*****************************************************************
C
C     THE FORMULA FOR UNSTABLE CONDITIONS GIVEN BELOW
C     FOR FSY BECOMES IDENTICAL OR NEARLY SO TO THE
C     FORMULA GIVEN BELOW FOR FSY FOR NEUTRAL CONDITIONS
C     AS CONDITIONS BECOME LESS UNSTABLE.
C     AS CONDITIONS BECOME CONVECTIVELY UNSTABLE, THE
C     FORMULA FOR FSY APPROXIMATES THE RESULTS FOUND
C     BY WILLIS AND DEARDORFF.
C
      XXO = 0.0
C
C     WE ZERO XXO EVERY TIME THROUGH BECAUSE IT IS USED
C     ONLY FOR STABLE CALCULATIONS.  IT LOOKS A BIT STRANGE
C     IF XXO DOES NOT VARY IN VALUE WHICH CAN HAPPEN IF
C     THE FIRST TIME THROUGH THE LOOP WE HAVE STABLE
C     CONDITIONS AND THE NEXT SEVERAL TIMES THROUGH WE
C     HAVE UNSTABLE CONDITIONS.  HENCE, ZERO XXO EVERY
C     TIME, LESS QUESTIONS THAT WAY.
C
      XTI = 0.0
C
C      AS WITH XXO SO ALSO WITH XTI.
C
      IF(TEST.LT.0.0) GO TO 300
C
C     WE TEST ON L HERE.  THIS IS OK SINCE
C     THE ROUTINE CAN HANDLE UNSTABLE DISPERSION
C     IN THE LATERAL DIMENSIONS RIGHT UP TO
C     NEUTRAL CONDITIONS.  THIS WAS NOT SO
C     FOR THE VERTICAL DISPERSION CALC. SINCE WE
C     'CALLED' UNSTABLE CONDITONS WITH -HL/L'S
C     OF 10 OR LESS 'NEUTRAL' IN SO FAR AS THE
C     ESTIMATION OF THE DISPERSION FUNCTION WAS
C     CONCERNED.
C
C*** MUST BE STABLE (DRAXLER,1976)
C
      XXO = U*1000.0
      FSY = 1.0/( 1.0+0.9*SQRT(X/XXO) )
      GO TO 400
```

```
C
C*** MUST BE UNSTABLE (NIEUWSTADT AND DUUREN,1979)
C
  300 CONTINUE
      C = 2.5
      D = 0.00133
      OTI = (C*USTAR/HL)*(1.0-D*HL*OL)**0.33333
      XTI = U/OTI
      FSY = 1.0/( 1.0+SQRT(X/XTI) )
  400 SYO = SA*X*FSY
C
C     HORIZONTAL WIND SHEAR EFFECTS (PASQUILL, 1976;1974)
C
      H1 = ZR + 2.15*SZ
      IF(TEST.GT.0.0) GO TO 420
C
C     WE HAVE AN UPPER LIMIT TO THE VERTICAL EXTENT OF
C     THE PLUME (H1) DURING UNSTABLE AND NEUTRAL CONDITIONS.
C     HOWEVER, DURING STABLE CONDITIONS THIS IS NOT SO.
C
      IF(H1.GT.HL) H1 = HL
  420 H2 = ZR - 2.15*SZ
      IF(H2.LT.0.0) H2 = 0.0
C
C     THE ABOVE CHECK KEEPS THE LOWER EXTENT OF THE
C     PLUME ABOVE THE GROUND.
C
      SY2 = 0.173*DTHETA*(H1 - H2)*X
      SY = SQRT(SYO*SYO+SY1*SY1+SY2*SY2)
C
C*************************************************************
C                                                            *
C               FINISH                                       *
C       LATERAL DISPERSION CALCULATIONS                      *
C                                                            *
C*************************************************************
      GO TO 1000
  900 CONTINUE
C
C     ERROR DETECTED IN SPECIFICATION OF THE INPUT
C
      FSZ = 0.0
      FSY = 0.0
      SZ = 0.0
      SY = 0.0
      RZR = 0.0
      IKEY = -1
 1000 RETURN
      END
```